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Modeling competition in natural gas markets

by

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ABSTRACT

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This dissertation consists of three chapters; each models competition in natural gas markets. These models provide insight into interactions between changes in market conditions/policies and market players' strategic behavior. In all three chapters, we apply our models to a natural gas trade network formed by using BP's Statistical Review of World Energy 2010 major trade flows.

In the first chapter, we develop a model for the world natural gas market where buyers and sellers are connected by a trading network. Each natural gas producer is a Cournot player with a fixed supply capacity. Each of them is also connected to a unique set of importing markets. We show that this constrained noncooperative Cournot game is a potential game and its potential function has a unique maximizer.

In the scenario analysis, we find that any exogenous change affecting Europe also has an effect in the Asia Pacific. The reason is that two big producers, Russia and the Middle East, are connected to both markets. We also find that a collusive agreement between Russia and the Middle East leads them to specialize in supply to markets based on their marginal costs of exporting natural gas.

The second chapter is devoted to analyzing the impacts of North American shale gas on the world natural gas market. To better represent the North American natural gas market, this chapter also allows for perfect competition in that market. We find

that North America exports natural gas when its supply curve is highly elastic and hence the domestic price impact of its exports is very small. Even so, the price impacts on the importing markets are substantial. We also find that shale gas development in North America decreases dominant producers' market power elsewhere in the world and hence decreases the incentive of any parties to form a natural gas cartel.

In the third chapter, we relax the assumption of fixed supply capacities and allow for natural gas producers to invest in their supply capacities. We assume a two period model with no uncertainty and show that there is a unique Cournot-Nash equilibrium and the open-loop Cournot-Nash equilibrium and closed-loop Cournot-Nash equilibrium investments coincide.

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Chapter 1

An imperfectly competitive model of the world natural gas market

1.1 Introduction

World natural gas production is concentrated in a small number of producers, the majority of which are state-owned companies. For instance, Russia's biggest natural gas producer, Gazprom, controls 70 percent of Russian natural gas reserves and produces 78 percent of all Russian natural gas.[1] Similarly, state-owned Sonatrach[2] dominates natural gas production and wholesale distribution in Algeria, while state-owned Sonelgaz controls retail distribution.[3] These state-owned companies have monopoly power in their domestic markets, but their ability to exploit it is limited since their actions are highly regulated by their governments.[4] Once they export natural gas via long distance pipeline or liquefied natural gas (LNG), they must compete with each other and sometimes with domestic suppliers in the European, Asian or North American markets. In contrast to supply, the demand for natural gas in these foreign markets is not as concentrated. In this chapter, we assume that natural gas consumers (e.g., utility service providers) do not have any bargaining power and are passive agents. We then analyze the strategic behavior of world natural gas producers[5] and examine

[1] See http://www.nord-stream.com/about-us/our-shareholders/

[2] See Oil and Gas Directory Middle East, 2011.

[3] Other examples include Qatar Petroleum in Qatar, Nigerian National Petroleum Corporation in Nigeria, National Gas Company of Trinidad and Tobago in Trinidad and Tobago, Pertamina in Indonesia and Petronas in Malaysia.

[4] For instance, natural gas prices in Russia are regulated by the Federal Tariff Service of the Russian Federation.

[5] To make the model tractable, we need to have small number of players. For that reason, we aggregate producers and consumers based on their geographic locations as well as their role in global

the impacts of exogenous changes on their behavior.

Long distance, and especially international, natural gas transportation infrastructure is expensive to construct and generally changes slowly. The high costs of developing large natural gas projects have also led to long-term contracts tying buyers to particular sellers. As a simplifying assumption, we consider a network structure where connections between buyers and sellers are fixed. In our model, a buyer and a seller must have a relationship, or "link" to trade. For instance, the Yamal pipeline from Russia to Europe is a link. The cost of building pipelines over long distances and the high cost of LNG shipment lead to differences in natural gas prices between regions. In our model, however, price discrimination by producers also contributes to price differentials between markets.

We modify Ilkilic (2010), who develops a bipartite network model for m markets and n firms in Cournot competition and analyzes how the structure of the network that connects suppliers with consumers affects the market outcome. Unlike Ilkilic (2010), we assume that each producer has a supply capacity constraint and solve for the equilibrium under this constraint. We show that our game can be represented as a potential game and solve for its equilibrium. We then consider various changes to the basic model in a number of scenarios.

As a consequence of imperfect competition within this given network structure, we find that any exogenous change affecting Europe has an offsetting effect in the Asia Pacific, as two big producers, Russia and the Middle East, are connected to both markets. We find that if Russia and the Middle East collude, Russia supplies Europe only whereas the Middle East supplies the Asia Pacific only. We also find that shale gas development in North America reduces natural gas producers' market power around the world.

Section 1.2 reviews the literature on strategic interactions among natural gas market players. In Section 1.3, we present an overview of the world natural gas

natural gas trade. For instance, we assume that Russia includes Armenia, Azerbaijan, Kazakhstan, Kyrgyzstan, Tajikistan, Turkmenistan, Uzbekistan and Russia.

market in 2009. In Section 1.4, we define our Cournot game model and solve for its unique Cournot-Nash equilibrium. In Section 1.5, we calibrate the Cournot game parameters based on trade volumes from the 2009 world natural gas network in BP's Statistical Review of World Energy 2010 and natural gas prices in 2009. Section 1.6 is devoted to analyzing different policy scenarios. The chapter concludes in Section 1.7.

1.2 Related literature

Strategic interactions among natural gas market players have been widely studied. Mathiesen et al. (1987) were the first to claim that the natural gas market is best described by a Cournot game, as the majority of natural gas trade is based on long-term, take-or-pay contracts. Later, Golombek et al. (1995 and 1998) studied the European natural gas market as a Cournot game. They analyzed the effects of liberalization in Europe by distinguishing between upstream and downstream agents and arguing that deregulation increased upstream competition while leaving downstream markets regulated. The cost parameters and elasticity parameters of Golombek et al. (1995) were disaggregated by country[6] and hence have been widely used. For instance, the GASTALE model by Boots et al. (2004) used the marginal cost parameters in Golombek et al. (1995), which was the first paper to apply the successive oligopoly model[7] in natural gas production and trading in a large-scale simulation. However, Golombek et al. (1995) make simplifying assumptions, such as requiring symmetry among traders and taking domestic supply to be exogenous.

Holz et al. (2006) developed a strategic model of European gas supply, GASMOD, as a two-stage game. In the first stage, upstream exporters are Stackelberg leaders

[6]They provide the price elasticities for Belgium, France, Netherlands, Italy, United Kingdom and West Germany and the cost parameters for Algeria, Commonwealth of Independent States, Netherlands, Norway and United Kingdom.

[7]This is a model in which the upstream natural gas producers supply to the downstream traders to serve consumers located in the foreign country.

over downstream domestic wholesale traders. In the second stage, downstream traders take the prices determined by the upstream exporters as given and compete with each other to supply domestic markets. In particular, unlike GASTALE, GASMOD assumes that domestic production is endogenous. However, neither GASTALE nor GASMOD considers the European natural gas market as a network.

Gabriel et al. (2005) using a mixed complementarity[8] approach developed an equilibrium model of natural gas markets. Their model not only covers multiple seasons and years; it also allows for many different sectors or agents: producers, storage operators, peak gas operators, pipeline operators, marketers, and consumers. Marketers and consumers interact strategically, while other trades are competitive. Marketers are price takers when purchasing natural gas from storage operators, pipeline operators, and peak gas operators but behave strategically when selling to end-use consumers. At any consumption node, there are several marketers who are connected to all four sectors; these sectors are residential, commercial, industrial and power. Their network graph is complete[9] and each marketer is only at one consumption node. Gabriel et al.'s (2005) model has been applied to North American, European and world natural gas markets.[10] In Gabriel et al.'s (2005) model the structure of the network is not the main consideration. In their model, the "network" has a special feature: a strategic player at any consumption node is connected to all markets at that node. Furthermore, each strategic player is connected to only one consumption node, and all players at a given node have the same objective function.[11] This simplifies the problem because there will be a representative strategic player on each

[8]A mixed complementarity model consists of set of simultaneous (linear or nonlinear) equations that are mix of strict equalities and inequalities, with each inequality linked to a bounded variable in a complementarity slackness condition (Rutherford 1995).

[9]Formally, their graph is a complete bipartite graph meaning that every node of the first set (marketers) is connected to every node of the second set (end use consumers).

[10]For its application to the North American natural gas market, see Gabriel et al. (2005), to the European natural gas market see Egging and Gabriel (2006), Egging et al. (2008) and Holz (2009) and to the world natural gas market see Egging et al. (2010).

[11]Each marketer is connected to same sources, pipeline operators, storage operators and peak gas operators and purchases the natural gas at the same price.

consumption node, and the Cournot equilibrium is symmetric.[12] By contrast, both in reality and also in our model, the market power of each natural gas producer depends on its ability to access markets, and different producers will supply a different set of several markets.

The application of oligopolistic market models by Gabriel et al. (2005) to study natural gas markets differs from ours in two more ways. Their method depends on the existence of a solution to a system of equations and inequalities that result from mixed complementarity (Kuhn-Tucker) conditions, whereas ours relies on a constrained function minimization procedure. We explicitly use the literature on network resource allocation problems with coupled constraints to show how one can derive a minimization procedure that is equivalent to the constrained noncooperative game.

The solution approach that we use, which was introduced by Monderer and Shapley (1997), is called potential games. It has been applied in the electrical engineering literature on wireless networks and communication network problems.[13] A final difference between our model and Gabriel et al. (2005) is the kind of equilibrium that they try to establish. They compute a Cournot-Nash equilibrium that would also satisfy a market clearing condition. We look for a coupled constraint[14] Cournot-Nash equilibrium, which is a more appropriate solution concept for the natural gas network problem. This is because each producer's value from supplying a given market depends on its own actions and on the actions of competitors who are connected to the same market. In addition, a firm's actions in one market depend on its actions in other markets to which it is connected.

In summary, our model captures the strategic interactions among the small number of natural gas producers in the world natural gas network.[15] Contrary to previous

[12]The best response function of a player will be the same as his competitors' best response functions due to the symmetry of the Cournot game.

[13]See Zhu (2008).

[14]A producer's supply to a market is constrained by its supply to several other markets.

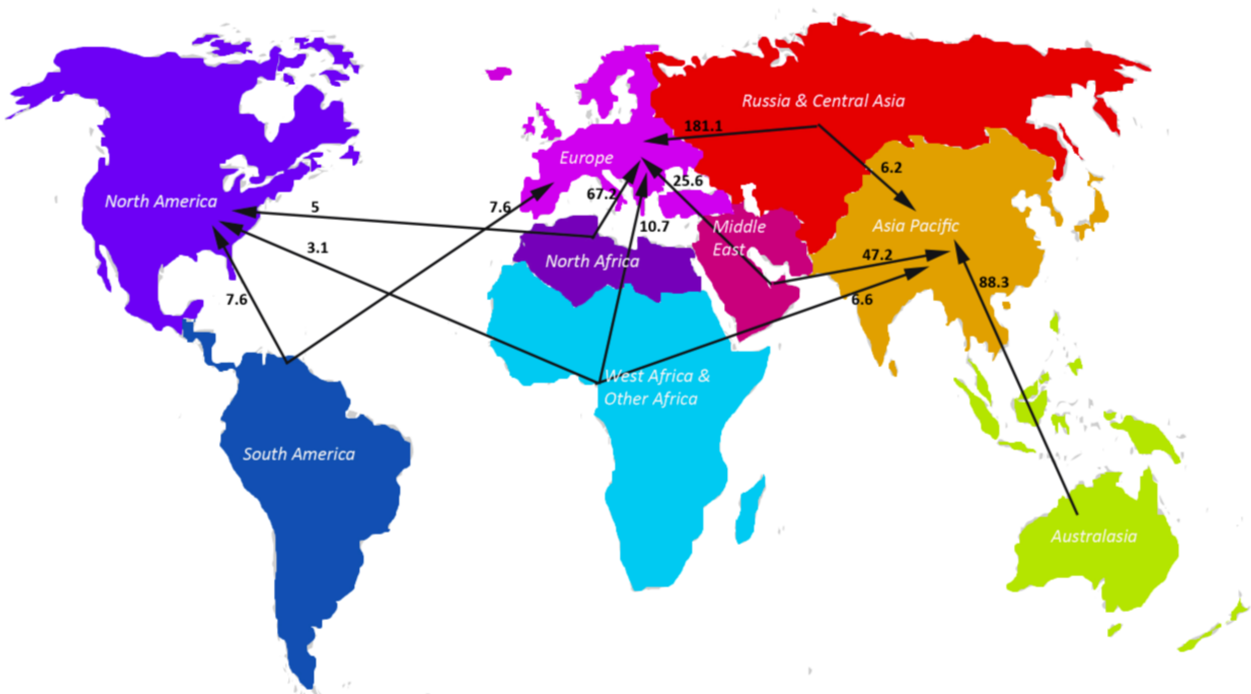
[15]The world natural gas network is based on the BP's Statistical Review of World Energy 2010's major trade flows and is not a complete graph. This means that not every producer is connected to

authors, we assume that the strategic players are heterogeneous in terms of their access to markets, their costs of exporting natural gas, and their supply capacities.

1.3 World natural gas market

Taking account of the strategic interaction between suppliers adds to the complexity of our model. To simplify, we therefore aggregate producers and consumers into a small number of regions and equilibrium trade flows as shown in the world map in Figure 1.1.

Figure 1.1 : Aggregated representation of producers and consumers and natural gas trade movements in 2009 (in Bcm)



Since each producer is connected to its domestic market, the number of producers and consumers is identical and in our case equals nine. In addition, six of the nine

every market.

producers are exporters, and three of the nine consumers are importers. Producers and consumers are ordered[16] as Europe,[17] North America,[18] Asia Pacific,[19] South America,[20] West Africa,[21] North Africa,[22] Russia,[23] Middle East[24] and Australasia.[25]

According to the BP's Statistical Review of World Energy 2010, in 2009, North America's total natural gas consumption was 828 billion cubic meters (Bcm) and total production was 812.95 Bcm. In 2009, North America imported 42 percent of its natural gas from Trinidad and Tobago and 29 percent from Egypt.

In 2009, Europe's total natural gas consumption was 580.3 Bcm and total production was 288.1 Bcm. The production-to-consumption ratio for Europe was 0.49; thus, more than 50 percent of the natural gas consumed in Europe in 2009 was imported. Russia was the largest supplier of natural gas to Europe, with a 62 percent share of imports. The Middle East's share in European natural gas imports was 8.8 percent and North Africa's share was 23.3 percent.

In 2009, Asia Pacific's total natural gas consumption was 394.4 Bcm and its total production was 246.1 Bcm. The production-to-consumption ratio for the Asia Pacific

[16] They are labeled according this order. Producers: Europe labeled as 1, North America labeled as 2, Asia Pacific labeled as 3, South America labeled as 4, West Africa labeled as 5, North Africa labeled as 6, Russia labeled as 7, Middle East labeled as 8, Australasia labeled as 9. Consumers are in the same order as producers and labeled the same.

[17] Europe includes Austria, Belarus, Belgium, Bosnia, Bulgaria, Croatia, Czech Republic, Estonia, France, Germany, Greece, Hungary, Ireland, Italy, Latvia, Lithuania, Luxembourg, Macedonia, Moldova, Netherlands, Poland, Portugal, Romania, Serbia, Slovakia, Slovenia, Spain, Sweden, Switzerland, Turkey, United Kingdom and Ukraine.

[18] North America includes Mexico, U.S. and Canada.

[19] Asia Pacific includes Bangladesh, China, india, Japan, Myanmar, Pakistan, South Korea, Taiwan, Thailand and Vietnam.

[20] South America includes Argentina, Bolivia, Brazil, Colombia, Peru, Trinidad and Tobago, Venezuela.

[21] West Africa includes Angola, Equatorial Guniea, Mozambique and Nigeria.

[22] North Africa includes Algeria, Egypt and Libya.

[23] Russia includes Armenia, Azerbaijan, Kazakhstan, Kyrgyzstan, Tajikistan, Turkmenistan, Uzbekistan and Russia.

[24] Middle East includes Iran, Israel, Kuwait, Lebanon, Oman, Saudi Arabia, Syria, Qatar, U.A.E., Yemen.

[25] Australasia includes Australia, Brunei, Indonesia, Malaysia, New Zealand, Philippines and Singapore.

was 0.62. Australasia supplied 59.5 percent of Asia Pacific's natural gas imports, making it Asia Pacific's largest supplier. The Middle East accounted for 31.8 percent of Asia Pacific's natural gas imports. Russia exported 6.2 Bcm of natural gas to the Asia Pacific in 2009, which was 3.7 percent of total imports. Before 2009, Russia had no natural gas exports to the Asia Pacific.

According to the BP's Statistical Review of World Energy in 2010, the U.S. Henry Hub natural gas price was 3.89 USD per million British thermal units (MMBtu). However, according to the OECD data on natural gas import costs, the U.S. LNG import cost was 4.52 USD per MMBtu. Due to our single price assumption for each region, the North American price in this model is 150 million USD per Bcm, which is approximately 4.18 USD per MMbtu.[26]

For the natural gas price in the Asia Pacific, we use LNG Japan price data reported by the BP's Statistical Review of World Energy in 2010, which is 9.06 USD per MMBtu. For natural gas price in the European market, we use the average of German import price, LNG and pipeline import prices for the European Union members provided by the OECD, which is 8.4 USD per MMbtu.

In our model, natural gas prices in the European and the Asia Pacific markets are close to each other and higher than the North American price. However, according to Figure 5 in Medlock (2012) the prices of natural gas at the U.S. Henry Hub, the UK National Balancing Point, the Platts Japan/Korea Marker were close before the Fukushima incident. We need to consider the historical natural gas price trends among these markets in our future research.

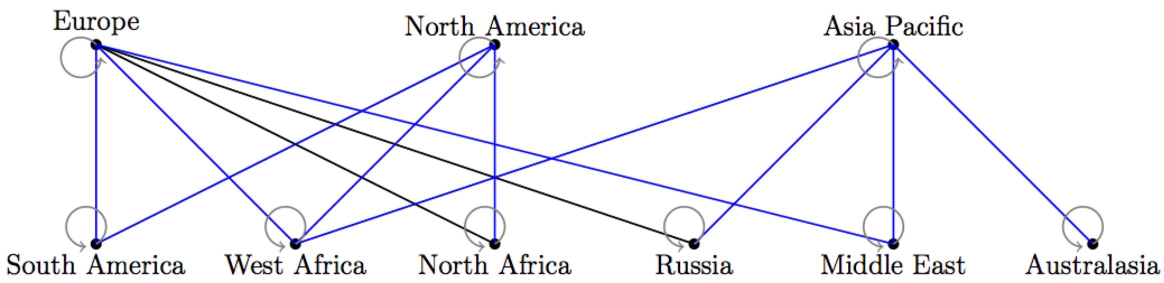
1.3.1 Schematic representation of the world natural gas trade

The world natural gas network formed using these statistics[27] is shown below.

[26] This price reflects the natural gas price in Canada, U.S. and Mexico.

[27] The blue lines indicate that the natural gas is transported via LNG and the black lines indicate that the natural gas is transported via pipeline. Half of the natural gas exports from North Africa to Europe are carried via LNG and half of them are carried via pipeline. Each producer is connected to its domestic market, which is indicated by gray circle.

Figure 1.2 : Schematic representation



1.4 Model

1.4.1 Notation

There[28] are m markets[29] $d_1, ..., d_m$ and n firms[30] $f_1, ..., f_n$. They are embedded in a network that links markets with firms, and firms can supply only to the markets to which they are connected. This network will be represented as a set, $g = \langle D \cup F, L \rangle$, of *nodes* formed by markets $D = d_1, ..., d_m$, and firms $F = f_1, ..., f_n$ and a set of *links* L, each link joining a market with a firm. A link from d_i to f_j will be denoted as (i, j). We say that a market d_i is *linked* to a firm f_j if f_j supplies natural gas to market d_i, using the link joining the two. We will use $(i, j) \in g$ meaning that d_i and f_j are connected in g.

A graph is *connected* if there exists a path connecting any two nodes of the graph while ignoring direction of physical flows. This concept is important because in a connected graph any change affecting one node will impact all other nodes.

$N_g(d_i)$ will denote the set of firms linked with d_i in $g = \langle D \cup F, L \rangle$. More formally:

$$N_g(d_i) = \{f_j \in F \text{ such that } (i, j) \in g\} \tag{1.1}$$

[28] We use the conventions set forth in Ilkilic (2010).

[29] We use terms "market", "consumer" and "buyer" interchangeably.

[30] We use terms "firm", "producer" and "seller" interchangeably.

and similarly $N_g(f_j)$ stands for the set of markets linked with f_j.

1.4.2 Cournot game

Given a graph g, each firm f_j maximizes its profit by supplying non-negative quantities to the markets in $N_g(f_j)$. Thus, the set of strategic players is the set of firms F. Let $q_{ij} \geq 0$ be the quantity supplied by firm f_j to the market d_i and Q_g be the vector of quantities supplied in graph g. Let $r(g)$ be the size of Q_g, and assume we list the supply q_{ij} above the supply q_{kl} when $j < l$ or when $j = l$ and $i < k$.

A simplified example might help the reader understand the general formulation. Consider a network with four producers,[31] four markets,[32] and seven links connecting them. Out of these four markets, two are importers, market 3 and market 4. Out of these four producers, two are exporters, producer 1 and producer 2.[33]

Natural gas carried from producer 1 to market 1 is denoted as q_{11}, from producer 1 to market 3 is denoted as q_{31}, from producer 1 to market 4 is denoted as q_{41}, from producer 2 to market 2 is denoted as q_{22}, from producer 2 to market 4 is denoted as q_{42}, from producer 3 to market 3 is denoted as q_{33} and from producer 4 to market 4 is denoted as q_{44}.

We write the vector of quantities supplied in this graph as:

$$Q_g = \begin{bmatrix} q_{11} & q_{31} & q_{41} & q_{22} & q_{42} & q_{33} & q_{44} \end{bmatrix}$$

We assume that markets have linear inverse demand functions. Given a market d_i and a flow vector Q_g the price, p_i, at d_i is

$$p_i(Q_g) = \alpha_i - \beta_i h_i, \tag{1.2}$$

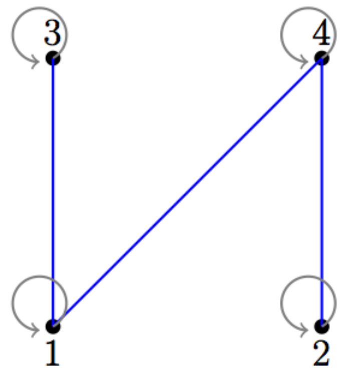
where α_i and β_i are positive constants and h_i is natural gas consumption in market d_i. More specifically h_i is

[31] They are labeled as 1, 2, 3 and 4.

[32] They are labeled as 1, 2, 3 and 4.

[33] The network graph is shown in Figure 1.3.

Figure 1.3 : A simplified example



$$h_i = \sum_{f_j \in N_g(d_i)} q_{ij}. \tag{1.3}$$

For example, the total consumption in market 4 in the simple network is $h_4 = q_{41} + q_{42} + q_{44}$, leading to linear inverse demand $p_4 = \alpha_4 - \beta_4(q_{41} + q_{42} + q_{44})$.

We assume that the natural gas producer has zero costs of production in the short run up to its production capacity, $\bar{S}_j$.[34] We also assume that the cost of exporting natural gas is linear. In the case of LNG, the export cost depends on the exporting country's liquefaction cost, the importing country's regasification cost and the distance traveled. In the case of pipeline transport, it depends on tariffs paid to transit countries and the length of the pipeline. For firm f_j the short-run total cost of supply therefore is

$$T_j(Q_g) = \sum_{d_i \in N_g(f_j) \setminus d_j} \tau_{ij} q_{ij} \tag{1.4}$$

[34]We assume that the supply capacity is fixed in the short-term because no new wells are drilled. Our main focus in this study is to capture short-run strategic interactions among the producers. In the third chapter, we change the model to a two-stage game. In the first stage, a producer chooses the level of investment for its supply. In the second stage, it decides how much to supply to each market to which it is connected.

where τ_{ij} is the marginal cost of exporting natural gas to market i.[35] If the natural gas is carried via LNG, τ_{ij} includes[36] the port-to-port cost of shipment, and the costs of liquefaction and regasification. If the natural gas is carried via pipeline, τ_{ij} includes tariffs paid to transit countries, the cost of fuel lost during transportation, and the cost of operations and maintenance of the pipeline.

Firm j's total supply is denoted as s_j:

$$s_j = \sum_{d_i \in N_g(f_j)} q_{ij} \tag{1.5}$$

where $s_j \leq \bar{S}_j$. Given a graph Q_g and a supply capacity of $\bar{S}_j$, firm j maximizes profit by choosing q_{ij}:[37]

$$\max_{q_{ij}} \pi_j = \max_{q_{ij}} \left\{ \sum_{d_i \in N_g(f_j)} \alpha_i q_{ij} - \sum_{d_i \in N_g(f_j)} \beta_i q_{ij}^2 - \sum_{d_i \in N_g(f_j)} \beta_i q_{ij} \sum_{f_k \in N_g(d_i) \setminus f_j} q_{ik} - \sum_{d_i \in N_g(f_j) \setminus d_j} \tau_{ij} q_{ij} \right\} \tag{1.6}$$

subject to

$$\sum_{d_i \in N_g(f_j)} q_{ij} \leq \bar{S}_j \tag{1.7a}$$

$$q_{ij} \geq 0 \quad \text{for all} \ \ (i, j) \in g \tag{1.7b}$$

We write the Kuhn-Tucker Lagrangian of firm j's maximization problem as

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[35] We assume that cost of exporting natural gas is proportional to the export volume.

[36] These costs are per unit of natural gas, that is one Bcm in this chapter.

[37] For the graph Q_g in the simple example, producer 1 maximizes

$$\max_{q_{11}, q_{31}, q_{41}} (\alpha_1 - \beta_1 q_{11}) q_{11} + (\alpha_3 - \beta_3 (q_{31} + q_{33})) q_{31} + (\alpha_4 - \beta_4 (q_{41} + q_{42} + q_{44})) q_{41} - \tau_{31} q_{31} - \tau_{41} q_{41}$$

$$\mathcal{L} = \sum_{d_i \in N_g(f_j)} \alpha_i q_{ij} - \sum_{d_i \in N_g(f_j)} \beta_i q_{ij} h_i - \sum_{d_i \in N_g(f_j) \backslash d_j} \tau_{ij} q_{ij} + \lambda_j \left(\bar{S}_j - \sum_{d_i \in N_g(f_j)} q_{ij} \right) + \sum_{d_i \in N_g(f_j)} \mu_{ij} q_{ij}. \tag{1.8}$$

Then there exists $\lambda_j^\star$ and $\mu_{ij}^\star$ such that $q_{ij}^\star$, $\lambda_j^\star$ and $\mu_{ij}^\star$ satisfy the following Kuhn-Tucker optimality conditions:

$$\frac{\partial \mathcal{L}}{\partial q_{ij}} = \alpha_i - \tau_{ij} - \beta_i \left(\sum_{f_k \in N_g(d_i) \backslash f_j} q_{ik} + 2q_{ij}^\star \right) - \lambda_j + \mu_j + \iota_{ij} = 0 \tag{1.9a}$$

$$\frac{\partial \mathcal{L}}{\partial \lambda_j} = \bar{S}_j - \sum_{d_i \in N_g(f_j)} q_{ij} \geq 0 \tag{1.9b}$$

$$\frac{\partial \mathcal{L}}{\partial \mu_{ij}} = q_{ij} \geq 0 \tag{1.9c}$$

$$\lambda_j \frac{\partial \mathcal{L}}{\partial \lambda_j} = \lambda_j \left(\bar{S}_j - \sum_{d_i \in N_g(f_j)} q_{ij} \right) = 0 \tag{1.9d}$$

$$\mu_{ij} \frac{\partial \mathcal{L}}{\partial \mu_{ij}} = \mu_{ij} q_{ij} = 0. \tag{1.9e}$$

We get the[38] Cournot-Nash equilibrium flow of $q_{ij}^\star$:

$$q_{ij}^\star = \begin{cases} \dfrac{\alpha_i - \tau_{ij} - \lambda_j - \beta_i \left(\sum_{f_k \in N_g(d_i) \backslash f_j} q_{ik} \right)}{2\beta_i} & \text{if } \frac{\partial \pi_j}{\partial q_{ij}} \Big|_{Q_g} \geq 0 \\ 0 & \text{if } \frac{\partial \pi_j}{\partial q_{ij}} \Big|_{Q_g} < 0 \end{cases} \tag{1.10}$$

The stylized representation of the current world natural gas market described above is a non-cooperative game with coupled payoff functions and coupled con-

[38] Ilkilic (2010) shows that the unconstrained Cournot game in a bipartite graph has a unique Nash equilibrium.

straints.[39] Although the Lagrangian multiplier theory is widely used to solve nonlinear mathematical programming problems with constraints,[40] it is not computationally convenient to apply it to our model. In particular, we have a large number of first-order conditions with inequality constraints (one for each producer) that need to be solved simultaneously. Instead, we use the potential game method. This involves re-casting our model as a single constrained optimization problem as if it were being solved by a centralized approach where there is a single agent who searches for the optimal solution given the constraints.[41]

Definition 1: Consider the Cournot game that we describe above with linear inverse demand function[42] and linear cost function. Define a function

$$\begin{aligned} P^\star(Q_g) = & \sum_{d_i \in N_g(f_j)} \alpha_i \left(\sum_{f_j \in N_g(d_i)} q_{ij} \right) - \sum_{d_i \in N_g(f_j)} \beta_i \left(\sum_{f_j \in N_g(d_i)} q_{ij}^2 \right) \\ & - \sum_{d_i \in N_g(f_j)} \beta_i \left(\sum_{1 \le j < k \le n} q_{ij} q_{ik} \right) - \sum_{f_j \in N_g(d_i)} \sum_{d_i \in N_g(f_j) \setminus d_j} \tau_{ij} q_{ij} \end{aligned} \tag{1.11}$$

subject to

$$\bar{S}_j \geq \sum_{d_i \in N_g(f_j)} q_{ij} \quad \text{for all } \; j \in F \tag{1.12}$$

and

[39] The coupling arises because producers have limited capacity of production to allocate to markets to which they are connected.

[40] Among others see Bertsekas (1998), Boyd and Vandeberghe (2004) and Bazaraa, Sherali and Shetty (1993).

[41] We note that this is a mathematical device only. We do not assume that there is a single world authority planning all natural gas trades. In particular, the optimization embeds the efficiency costs of oligopoly. Presumably, if there were a single centralized planner, that agent would choose an efficient outcome.

[42] Monderer and Shapley (1994) define a potential function for a Cournot game with linear inverse demand function. We adapt their functional form to our network Cournot game.

$$q_{ij} \geq 0 \quad \text{for all } \ (i,j) \in g. \tag{1.13}$$

It can be verified that for every link from firm j to market i, that is q_{ij}, and for every link that is not from firm j to market i, that is q_{-ij}, $P^{\star}(Q_g)$[43] satisfies

$$\pi_j(q_{ij}, q_{-ij}) - \pi_j(x_{ij}, q_{-ij}) = P^{\star}(q_{ij}, q_{-ij}) - P^{\star}(x_{ij}, q_{-ij}) \tag{1.14}$$

A function $P^{\star}$ satisfying (1.14) is called a potential function which requires

$$\frac{\partial \pi_j}{\partial q_{ij}} = \frac{\partial P^{\star}}{\partial q_{ij}} \quad \text{for all } \ (i,j) \in g \tag{1.15}$$

Theorem 1: The solution to the potential game[44] defined in (1.11) subject to constraints defined in (1.12) and (1.13) is unique:

$$\max_{q_{ij}} P^{\star}(Q_g) \quad \text{for all } \ (i,j) \in g \tag{1.16}$$

subject to (1.12) and (1.13).

Proof: See Section (1.8.1).

Theorem 2: The Nash equilibrium of the potential game with constraints defined in (1.16) and the Nash equilibrium of the noncooperative Cournot game with constraints defined in (1.6) are the same.

Proof of Theorem 2: Let Q'_g be the optimal solution to (1.16). Since Q'_g minimizes $\mathcal{L}_{P^{\star}}$, Q'_g minimizes $\mathcal{L}_j$ for each player j. Therefore, Q'_g is a Nash equilibrium to the constrained noncooperative game.[45]

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[43]Here Q_g is the vector of quantities in graph g.

[44]We know that the optimization problem defined in (1.11) is a potential game because it satisfies (1.14).

[45]Monderer and Shapley (1996) say that if a game that possesses an ordinal potential (the network game introduced in this chapter is an exact potential game, which is a subset of ordinal potential game) is called an ordinal potential game. Clearly, the pure strategy equilibrium set of the Cournot game coincides with the pure-strategy equilibrium set of the game in which every firm's profit is given by ordinal potential.

1.5 Calibration

In order to quantitatively evaluate different policy scenarios, we first need to calibrate the theoretical model. To calibrate the model parameters, we use the production, consumption, price and trade flow data in 2009. The price data is obtained from International Energy Agency's (IEA) website and other country websites. The data on production, consumption, and trade flows are obtained from BP's Statistical Review of World Energy 2010.

For calibration, we use the first order conditions of our model.

Example: South America's producer, labeled as 4, aims to [46]

$$\max_{q_{14},q_{24},q_{44}} \Pi_4(Q_g) = \max_{q_{14},q_{24},q_{44}} \{p_1 q_{14} + p_2 q_{24} + p_4 q_{44} - \tau_{14} q_{14} - \tau_{24} q_{24}\} \tag{1.17}$$

subject to

$$q_{14} + q_{24} + q_{44} \leq \bar{S}_4 \quad \text{and} \quad q_{14}, q_{24}, q_{44} \geq 0. \tag{1.18}$$

By considering the links that carry positive flows[47] in equilibrium, we get the first order conditions as:

q_{14}:

$$\alpha_1 - 2\beta_1 q_{14} - \beta_1(q_{11} + q_{15} + q_{16} + q_{17} + q_{18}) - \tau_{14} - \lambda_4 - \mu_{14} = 0 \tag{1.19}$$

q_{24}:

$$\alpha_2 - 2\beta_2 q_{24} - \beta_2(q_{22} + q_{25} + q_{26}) - \tau_{24} - \lambda_4 - \mu_{24} = 0 \tag{1.20}$$

[46] As an identifying assumption, we set that the cost of transporting natural gas to the domestic market as zero.

[47] According to Ilkilic (2010) links that carry zero flows in equilibrium have no role in determining the equilibrium.

q_{44}:

$$\alpha_4 - 2\beta_4 q_{44} - \lambda_4 - \mu_{44} = 0. \tag{1.21}$$

We assume an interior solution for the capacity constraint,[48] $q^\star_{14} + q^\star_{24} + q^\star_{44} < \bar{S}_4$, this requires $\lambda_4 = 0$.

We apply the same equilibrium condition to each producer from 1 to 9, and get twenty one equations.[49] The equilibrium price[50] in each market is denoted as $\hat{p}_i$.[51]

Insert Table (1.1) here.

We have 30 unknowns[52] and 30 equations to solve for these parameters. We substitute natural gas production, consumption, trade flow and price data in 2009 into these equations and calculate the parameters.

Insert Table (1.2) here.

Our network parameters are consistent with the world natural gas market experience in 2009. For instance, although the distance between Russia and the Asia Pacific is the shortest, the marginal cost of exporting natural gas from Russia and the Asia Pacific is the highest. This is due to Russia's limited natural gas production and liquefaction capacities on Sakhalin Island. On the other hand, the marginal cost of exporting natural gas from Russia to Europe is the lowest. This result is consistent with the Rice World Gas Trade Model's[53] (RWGTM) pipeline cost estimate of the

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[48] We make this assumption only when calibrating the parameters. This assumption is realistic especially in 2009, where due to the global recession, producers had excess supply capacities. When analyzing alternative scenarios we do not impose this assumption.

[49] For the rest of the equations see Appendix (1.8.2).

[50] We use linear inverse demand, which is defined in (1.2).

[51] Natural gas import prices are usually different for each importer and this price differs from the domestic producer's price. However, our model assumes that there is a single price of natural gas in each region, which is determined by the total supply of producers connected to that region.

[52] These unknowns are α_i, β_i where $i = 1, ..., 9$ and τ_{ij} where there are twelve (i, j) pairs in the world natural gas network graph.

[53] The Rice World Gas Trade Model is a tool for examining the effects of economic and political influences on the global natural gas market within a framework grounded in geologic data and economic theory.

Yamal pipeline.[54]

As depicted in Figure 1.4, North America has the most elastic demand curve among the importers. This could be because North America has more alternatives to natural gas than Europe and Asia Pacific.

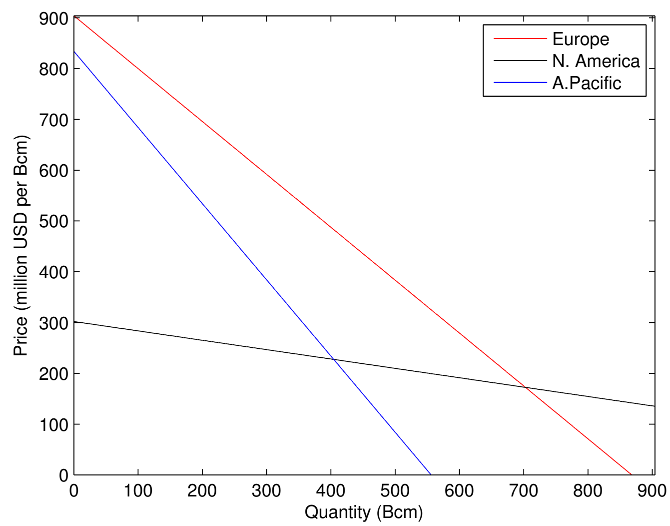

Figure 1.4 : Demand Curve of Europe, North America and the Asia Pacific, based on our calibration.

1.6 Scenario analysis

In this section we analyze various scenarios[55] in the world natural gas market by changing the model's parameters and/or capacity constraints exogenously. With each of these changes we optimize a new potential function subject to a new set of constraints. We use the sequential quadratic programming algorithm to solve for the

[54] According to the RWGTM, the marginal cost of carrying natural gas (tariffs plus the fuel cost plus the operating and maintenance of the pipeline) from Yamal to Germany (through Belarus and Poland) is 74.9 million USD per Bcm.

[55] Equilibrium trade flows under each scenario are provided in Table (1.3).

constrained optimum.[56]

1.6.1 Scenario I: Increased competition between Russia and the Middle East

According to BP's Statistical Review of World Energy 1997, about 84 percent of European natural gas imports came from Russia. Even though there has been a significant decline[57] in LNG transport costs since then, by 2009 Russia was still the biggest external supplier of European natural gas, with a share of 78 percent.[58] However, Russia's dominance in the European market is threatened by developments in Qatar,[59] and the concomitant doubling in LNG import capability in Europe since 2000.[60] From 2000 to 2009, Middle East exports to Europe increased from 0.84 Bcm to 25.6 Bcm. Nevertheless, Russia's dominance of the European natural gas market persists.

To reduce Europe's dependence on Russian natural gas, an alternative pipeline route, Nabucco, was proposed a decade ago. The goal was to connect European consumers to natural gas resources in the Caspian Sea area. Unlike the Nabucco pipeline project as originally proposed, we assume that a Nabucco pipeline would connect the Middle East and Europe.[61]

We incorporate this scenario in our model by using the RWGTM's cost estima-

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[56] More specifically, we use fmincon from MATLAB's optimization toolbox, which finds the minimum of a constrained nonlinear multivariable function. To obtain the maximum, we minimize (-1) times the potential function.

[57] For more details see The Global Liquefied Natural Gas Market: Status and Outlook, 2003.

[58] These claims are based on calculations using BP's Statistical Review of World Energy 2010's natural gas trade data.

[59] According to Dargin (2007), Qatar became the world's leading LNG exporter in 2006.

[60] See Medlock, Jaffe and Hartley (2011).

[61] In our model, countries around the Caspian Sea are considered to be part of the Russia super-region. Since the original Nabucco pipeline was proposed, analysts have questioned whether reserves in the Caspian region are not large enough to cover the capital cost of building a pipeline to Europe. On the other hand, developments in Iraq in particular have raised the possibility that the Middle East could become a large supplier of natural gas to Europe via pipeline.

tion[62] for Nabucco. The Nabucco route is assumed to go from Iraq to Istanbul, Istanbul to Bulgaria, then Bulgaria to Austria. We get the marginal cost of exporting to Europe by taking the weighted average of marginal costs of exporting natural gas to Europe via Nabucco and via LNG.[63]

We assume that 20 percent[64] of natural gas from the Middle East to Europe is carried via Nabucco and 80 percent is carried via LNG. With 20 percent via pipeline, the marginal cost of exporting one Bcm of natural gas from the Middle East to Europe decreases to 235.97 million USD. With this reduction, the Middle East increases its supply to Europe from 25.6 Bcm to 48.32 Bcm; decreases its supply to the Asia Pacific from 47.20 Bcm to 41.4 Bcm; and decreases its domestic market supply from 345.54 Bcm to 328.67 Bcm. The Middle East's share in Europe's natural gas imports increases from 3.09 percent to 5.83 percent. Europe's share in the Middle East's total production increases from 6.11 percent to 11.5 percent. When Nabucco is built, there will be more competition in the European market for all producers that are connected to it: Europe, South America, West Africa, North Africa, and Russia. They will decrease their supply to Europe to avoid further decline in the equilibrium natural gas price in Europe.

Contrary to the effects in Europe, the decline in supply from the Middle East will result in less competition in the Asia Pacific. As a result, West Africa, Russia, and Australasia will increase their supplies to the Asia Pacific. In the equilibrium, Russia's supply to the Asia Pacific increases from 6.2 Bcm to 7.9 Bcm; West Africa's supply to the Asia Pacific increases from 6.59 Bcm to 8.31 Bcm; and Australasia's supply to the Asia Pacific increases from 88.29 Bcm to 88.94 Bcm. The increase in

[62] We consider tariffs paid to transit countries plus the operating and maintenance costs. We ignore the capital cost as it is another decision problem which is beyond the motivation of this chapter.

[63] The cost of exporting natural gas via LNG is calibrated in the previous section.

[64] We also look at different scenarios such as: 30 percent is carried via pipeline and 70 percent is carried via LNG, and 50 percent is carried via pipeline and 50 percent is carried via LNG. The sign of changes in these scenarios are the same as in the case where 20 percent is carried via pipeline and 80 percent is carried via LNG, but the magnitudes are different. For instance, Middle East's supply to Europe is bigger when 50 percent of its exports are carried via pipeline.

supply from Russia and West Africa is greater than the increase from Australasia because the former two regions reap larger marginal profits than does Australasia from supplying the Asia Pacific region.

Under this scenario, equilibrium total supply to Europe goes from 580.3 Bcm to 584.9 Bcm, which decreases the equilibrium price in Europe from 300 million USD per Bcm to 296 million USD per Bcm. On the other hand, equilibrium total supply to the Asia Pacific declines from 394.39 Bcm to 392.96 Bcm, which increases the equilibrium price in the Asia Pacific from 320 million USD per Bcm to 322 million USD per Bcm. Neither the equilibrium price nor the equilibrium consumption changes in North America, since there is no change in the equilibrium supply to it.

Under this scenario, profits of the Asia Pacific, Middle East, and Australasia producers increase. Profits of North American producers stay the same and the profits of the remaining producers decrease. Both the Asia Pacific and Australasia have higher profits due to the increase in equilibrium price in the Asia Pacific. Europe, South America, West Africa, North Africa, and Russia have lower profits since their market shares, as well as the equilibrium price in Europe, decline.

In a variant of the Nabucco case we simply increase Middle East supply capacity by 10 percent while leaving transport costs to Europe unchanged. In the new equilibrium, the Middle East increases its supply to Europe without decreasing supply to the Asia Pacific or its domestic market.

1.6.2 Scenario II: Decreased competition between Russia and the Middle East

This scenario assumes that Russia and the Middle East collude to maximize their joint profits. Via such collusion they increase their market power in both markets they share and, hence, their joint profits increase.

Given the natural gas network we had in 2009, suppose Russia and the Middle

East[65] merge to maximize their joint profit, which is:

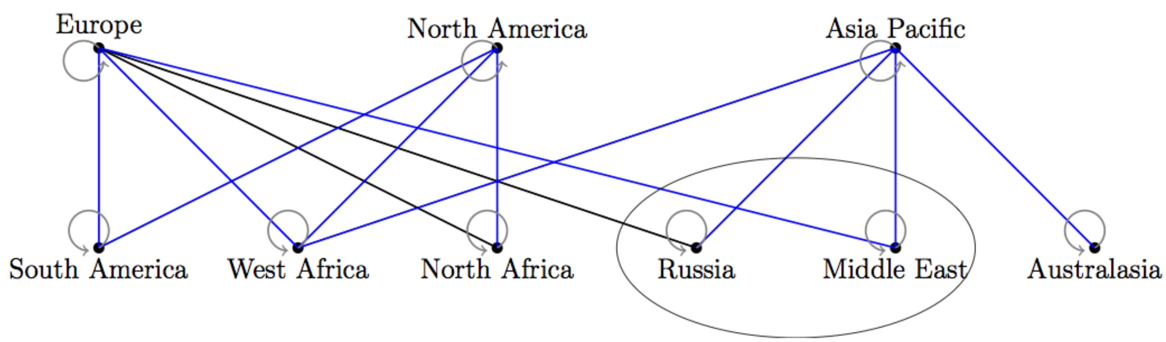
$$\Pi_{78}(Q_g) = \alpha_1(q_{17}+q_{18}) - \beta_1(q_{11}+q_{14}+q_{15}+q_{16}+q_{17}+q_{18})(q_{17}+q_{18}) + \alpha_3(q_{37}+q_{38}) - \beta_3(q_{33}+q_{35}+q_{37}+q_{38}+q_{39})(q_{37}+q_{38}) - \tau_{17}q_{17} - \tau_{18}q_{18} - \tau_{37}q_{37} - \tau_{38}q_{38} \tag{1.22}$$

subject to

$$q_{17}+q_{37}+q_{77} \leq \bar{S}_7\,, \quad q_{18}+q_{38}+q_{88} \leq \bar{S}_8 \quad \text{and} \quad q_{17},\, q_{37},\, q_{77},\, q_{18},\, q_{38},\, q_{88} \geq 0. \tag{1.23}$$

The graph of this new network is:

Figure 1.5 : Schematic representation after Russia and the Middle East merger



We optimize the new potential function subject to supply constraints. After the merger, Russia and the Middle East reduce their combined output and their equilibrium supplies to each of the markets that they share, Europe and the Asia Pacific. The new equilibrium outcome is that the links from Russia to the Asia Pacific and from the Middle East to Europe carry zero flows, meaning that Russia specializes in the European market and the Middle East specializes in the Asia Pacific. This occurs because Russia has a lower marginal cost of exporting natural gas to Europe,

[65] We label Russia and the Middle East after the merger as 78.

while the Middle East has a lower marginal cost of exporting natural gas to the Asia Pacific.

The equilibrium supply of Russia and the Middle East to Europe decreases to 187.75 Bcm. The pre-merger supply from Russia to Europe was 181.1 Bcm and from the Middle East to Europe was 25.6 Bcm. Similarly, the equilibrium supply of Russia and the Middle East to the Asia Pacific is 49.84 Bcm after the merger. The pre-merger supply from Russia to the Asia Pacific was 6.2 Bcm and from the Middle East to the Asia Pacific was 47.19 Bcm. As a result of the collusion, prices rise in both Europe and the Asia Pacific. In the new equilibrium, total supply to Europe decreases from 580.3 Bcm to 573.54 Bcm, which increases the equilibrium price from 300 million USD per Bcm to 307.03 million USD per Bcm. In the new equilibrium, total supply to the Asia Pacific decreases from 394.39 Bcm to 391.74 Bcm, which increases the equilibrium price in the Asia Pacific from 320 million USD per Bcm to 323.2 million USD per Bcm.

As these equilibrium outcomes indicate, the attempt to exploit consumers in Europe and the Asia Pacific via collusion is thwarted to some extent by other suppliers. South America, West Africa, and North Africa all increase their supply to Europe by decreasing their supply to North America and to their domestic markets. For instance, West Africa decreases its equilibrium supply to North America from 3.1 Bcm to zero and decreases its equilibrium supply to the Asia Pacific from 6.6 Bcm to 6.51 Bcm. Similarly, North Africa decreases its supply to North America from 5 Bcm to 1.46 Bcm in order to increase its supply to Europe. After the collusive merger, the marginal profit of supplying to North America declines for all producers that are connected to it.

The equilibrium prices after the merger increase in each market except Russia and the Middle East[66] due to a decline in equilibrium supply. For instance, the equilibrium supply to North America declines from 828.7 Bcm to 818.86 Bcm, and the equilibrium

[66] In the latter two markets they remain at the pre-merger level.

price increases from 150 million USD per Bcm to 151.816 million USD per Bcm.

The joint profit of Russia and the Middle East after the merger increases by 2.18 billion USD. Although this provides an incentive for Russia and the Middle East to merge, it is a weak one. Furthermore, the stability of such a collusion is hard to maintain since both parties would retain an incentive to "cheat" by reneging on a commitment to refrain from selling to their partner's exclusive market area.

As a further modification to the Russia and Middle East collusion, we increase North America's supply capacity by 5 percent and increase the marginal cost of exporting natural gas to it by 2 percent.[67] These changes make, the merger of Russia and the Middle East even less profitable (compared to no shale with collusion scenario). Due to the decline in import demand of North America, all exporters that are connected to it move their resources to other markets, that is, Europe, the Asia Pacific, and their domestic markets. Hence, Russia and Middle East's market power is reduced.

1.6.3 Scenario III: An increase in Asia Pacific's natural gas demand

According to the IEA's 2010 World Energy Outlook, China's demand for natural gas has recently grown faster than demand in any other region. In fact, it is projected to grow at an average of almost 6 percent per year 2008-2035. The IEA report projects that from 2008 to 2015, Asia's demand will grow from 341 Bcm to 497 Bcm a year.

In addition to demand growth from China and India, Japan's demand for natural gas has increased after the Fukushima nuclear disaster. According to the IEA's 2011 World Energy Outlook, the Fukushima disaster could lead to a 15 percent fall in world nuclear power generation by 2035, when power demand may be 3.1 percent higher. This will raise gas-fired power generation along with other types of generation. The incremental demand for LNG in Japan's power sector in 2011 was expected to be 11 Bcm, according to the IEA report.

[67] These changes are a simple way of representing the depressing effects on North American imports of increased shale gas production.

The demand increase expected in the 2011 report is incorporated in our model by increasing the choke price in the Asia Pacific by 5 percent. With a 5 percent increase in the choke price, natural gas demand in the Asia Pacific increases by 32.02 Bcm, which corresponds to an 8.12 percent increase in demand at the 2009 price in the Asia Pacific. All of the producers that are connected to the Asia Pacific respond to the demand increase by shifting supply from other markets to the Asia Pacific.

West Africa increases its supply to the Asia Pacific from 6.59 Bcm to 12.25 Bcm, decreases its supply to Europe from 10.7 Bcm to 8.34 Bcm and stops supplying North America in equilibrium. This is because the marginal profit of supplying to the Asia Pacific is greater than the marginal profit of supplying to Europe, North America, and to West Africa's domestic market.

Similarly, an upward shift in Asia Pacific's demand curve increases Russia's supply to the Asia Pacific from 6.2 Bcm to 13.65 Bcm and decreases Russia's supply to its domestic market from 485.43 Bcm to 478.13 Bcm. Russia thus meets most of its additional supply to the Asia Pacific from supplies to its domestic market. The share of Russia's supply to the Asia Pacific over its total production increases from 0.9 percent to 2 percent, whereas the share of its domestic market decreases from 72.2 percent to 71.1 percent.

With a 5 percent increase in Asia Pacific's choke price, the Middle East's supply to the Asia Pacific increases from 47.19 Bcm to 53.78 Bcm. This is achieved via a 5.34 Bcm cut in supply to its domestic market and a 1.19 Bcm reduction in its supply to Europe. The Asia Pacific's total consumption of natural gas from the Middle East increases by 0.9 percent.

As a result of an increase in Asia Pacific's demand, West Africa, Russia, and the Middle East decrease their supply to Europe, which makes the European market more attractive for South America and North Africa. In response, they decrease their supply to North America and their domestic markets.

The total consumption in each region except the Asia Pacific declines. The equilibrium price in these regions increases due to decline in equilibrium supply. Although the equilibrium supply to the Asia Pacific increases, the equilibrium price also increases due to the shift in demand for natural gas. Profits of each producer increase due to the increase in natural gas prices.

1.6.4 Scenario IV: Increase in importers' natural gas demand

We assume that the choke price in Europe, North America and the Asia Pacific increases by 2 percent. With this change, natural gas demand at 2009 prices increases by 3.02 percent in Europe, by 4.24 percent in North America, and by 3.3 percent in the Asia Pacific. We analyze world natural gas demand increase in two cases: with shale gas and without shale gas.

Without shale gas:

Following the increase in the choke price in importing countries, the marginal profit of exporting increases for each producer. Thus, they reduce their equilibrium supply to their domestic markets and increase their supply to abroad.

In the 2009 world natural gas trade network, West Africa is the only producer connected to all three importing markets. Under this scenario, West Africa increases its supply to Europe and the Asia Pacific and decreases its supply to North America, even though the demand in all three markets increases.

On the other hand, the rest of the producers increase their exports by decreasing their supply to their domestic markets. For instance, South America increases its equilibrium supply to Europe from 7.6 Bcm to 8.82 Bcm, increases its equilibrium supply to North America from 7.6 Bcm to 9.21 Bcm, and decreases its equilibrium supply to its domestic market from 134.7 Bcm to 131.86 Bcm.

Similarly, North Africa increases its equilibrium supply to Europe from 67.2 Bcm to 68.28 Bcm, and increases its equilibrium supply to North America from 4.99 Bcm to 5.85 Bcm.

Under this scenario, total supplies to Europe, North America, and the Asia Pacific increase. Due to the increase in demand, the equilibrium price in Europe, North America, and the Asia Pacific increases.[68]

The profit of each producer increases as the equilibrium prices in their domestic markets and abroad increase.

With shale gas:

We assume that, simultaneous with the demand increase, North America's supply capacity increases by 5 percent and the marginal cost of exporting natural gas to North America increases by 2 percent. These exogenous changes mimic the decrease in the import demand of North America resulting from the exploitation of shale gas.

Due to the decrease in North American import demand the producers connected to North America move their exports to Europe and the Asia Pacific. The market share of Russia, the Middle East, and Australasia decrease in Europe and the Asia Pacific compared to the no shale gas scenario. For instance, if there is no shale gas, Russia supplies 185.58 Bcm to Europe but with shale gas, it supplies 184.05 Bcm. Similarly, the Middle East's supply to Europe declines from 28.92 Bcm to 27.67 Bcm. The impact of shale gas development in North America is greater in Europe than in the Asia Pacific since Europe and North America share more producers than the Asia Pacific and North America.

Equilibrium prices in Europe, North America, and the Asia Pacific are lower than the ones under the no shale gas production scenario. However, they are higher than the prices in 2009 due to the concomitant increase in natural gas demand. With shale gas development, North America's total consumption increases from 830.3 Bcm to 840.92 Bcm and imports decrease from 17.3 Bcm to 7.3 Bcm. North America's

[68]The equilibrium price in Europe increases from 300 million USD per Bcm to 306.73 million USD per Bcm, in North America it increases from 150 million USD per Bcm to 155.76 million USD per Bcm and in the Asia Pacific it increases from 320 million USD per Bcm to 326.38 million USD per Bcm.

profit increases after the shale gas development while the rest of the producers make losses compared to a demand increase with no shale gas scenario.

1.6.5 Scenario V: Russia to China pipeline

In this scenario we assume that Western Siberia and China are connected through a pipeline. To incorporate this scenario into our model we use the Rice World Gas Trade Model's cost estimations for pipeline routes from Surgut to Urengoy, from West Siberia to China, from West China to Xian, and from Xian to Shanghai. We assume that 30 percent of natural gas from Russia to Asia Pacific is carried via pipeline and 70 percent is carried via LNG. We get the marginal costs of exporting natural gas to the Asia Pacific by taking the weighted average of marginal costs of exporting natural gas to the Asia Pacific via pipeline and via LNG.

If 30 percent of natural gas is carried via pipeline, then the marginal cost of exporting one Bcm of natural gas from Russia to the Asia Pacific decreases to 237.22 USD. With this reduction, Russia increases its supply to the Asia Pacific from 6.19 Bcm to 47.83 Bcm, decreases its supply to Europe from 181.1 Bcm to 173.86 Bcm, and decreases its domestic market supply from 485.43 Bcm to 451.09 Bcm. Russia's share in the Asia Pacific's imports increases from 3.7 percent to 32.25 percent. This increases the competition in the Asia Pacific for the Asia Pacific, West Africa, the Middle East, and Australasia. For that reason, they decrease their supply to the Asia Pacific. For instance, in the new equilibrium Australasia's supply to the Asia Pacific decreases from 88.29 Bcm to 79.54 Bcm and West Africa does not supply the Asia Pacific.

Under this scenario, the Middle East's equilibrium supply to the Asia Pacific decreases from 47.19 Bcm to 38.44 Bcm and its supply to Europe increases from 25.6 Bcm to 27.2 Bcm.

The decrease in natural gas supply from Russia to Europe decreases the competition in the European market. As a result, the marginal profit of supplying to Europe increases for South America, West Africa, North Africa, and the Middle East. For instance, South America's supply to Europe increases from 7.6 Bcm to 8.82 Bcm, West Africa's supply to Europe increases from 10.7 Bcm to 12.3 Bcm, and North Africa's supply to Europe increases from 67.2 Bcm to 68.42 Bcm. South America and North Africa decrease their supply to North America and to their domestic markets in order to supply more to Europe.

Under this scenario, equilibrium total supply to Europe decreases from 580.3 Bcm to 578.7 Bcm, which increases the equilibrium price in Europe from 300 million USD per Bcm to 301.66 million USD per Bcm. However, the equilibrium total supply to the Asia Pacific increases from 394.4 Bcm to 403.15 Bcm, and this decreases the equilibrium price in the Asia Pacific from 320 million USD per Bcm to 308.61 million USD per Bcm. The equilibrium price in North America increases by 0.19 million USD per Bcm as the total supply to North America decreases by 1.06 Bcm. Russia's natural gas prices in its domestic market increase due to the increased exports. They produce a decline in domestic market consumption.

Russia's profits increase from 65.76 billion USD to 67.89 billion USD, North America's profits increase from 121.95 billion USD to 122.11 billion USD, and the rest of the producers' profits decrease. The Asia Pacific has the biggest profit loss among remaining producers[69] because both its market share in the Asia Pacific and the equilibrium natural gas price in its domestic market decline.

1.6.6 Scenario VI: Russia to China pipeline and Nabucco

In this section, we consider the case where Russia has a pipeline connection to the Asia Pacific and the Middle East has a pipeline connection to Europe. We compare these results with the results in 1.6.1.[70]

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[69] Its profits decrease by 7.5 billion USD.

[70] That is the scenario where we considered increased competition between Russia and Middle East in the European market via a pipeline connection from the Middle East to Europe.

Under this scenario, Russia increases supply to the Asia Pacific to 49.35 Bcm, which is 1.51 Bcm higher than the scenario where there is only a Russia-China pipeline[71] and 41.43 Bcm higher than the case where there is only Nabucco. Russia's equilibrium supply to Europe is 169.93 Bcm; however, its equilibrium supply to Europe is 177.31 Bcm under the scenario where there is only Nabucco. The Middle East increases its supply to Europe to 51.54 Bcm, which is 3.22 Bcm higher than the scenario where there is only Nabucco and 24.35 Bcm higher than the scenario where there is only a Russian pipeline.

Under this scenario, Europe's, South America's, and North Africa's supplies to Europe are higher than the scenario where there is only Nabucco. in equilibrium, West Africa does not supply to the Asia Pacific.

Under this scenario, Russia's profits are higher than its profits in 2009 and its profits when there is Nabucco only. It is in Russia's best interest to have a Russia-China pipeline when there is Nabucco.

1.7 Conclusions

This chapter presented a network model of the world natural gas market that consists of consumers, producers (which are represented as strategic Cournot players), and links connecting them. We calibrated the model parameters using natural gas consumption, production, trade, and price data in 2009. The model allowed us to quantify the strategic interactions among natural gas producers.

We find that if a natural gas producer has access[72] to a market then its market power at that market depends on its production capacity and its costs of exporting natural gas. For instance, Russia's market power in the Asia Pacific is low because of its high costs of exporting natural gas from Sakhalin island to Japan.

[71] The decrease in the Middle East's cost of exporting natural gas to Europe will increase Middle East's exports to Europe making the European market more competitive for Russia.

[72] If it does not have access then its market power is zero.

We also find that any exogenous change affecting one market impacts all other markets. The size of the impact on any one market also declines as the number of links connecting markets increases.

Although the establishment of a single (or a reference) price for natural gas is difficult to achieve due to the high cost of transport and long-term contracts over 20-25 years with prices indexed to oil, our model confirms that with the developments in LNG technology and growing diversity in supply sources and new demand sinks, natural gas has been evolving into a global commodity with some convergence in regional prices.

1.8 Appendix

1.8.1 Proof of Theorem 1

The proof follows Zhu (2008) which introduces the game as minimization problem. *Corollary 2.8* in Zhu (2008) says that every strictly convex potential game admits a unique equilibrium. We therefore show that $(-1)\times$ our potential function, $(-1)\times P^{\star}(Q_g) = f(Q_g)$ is strictly convex in each q_{ij} for all $(i,j) \in N_g$.

Now $f : \mathcal{R}^{r(g)} \to \mathcal{R}$ where $r(g)$ is the size of Q_g, which is the number of links in the network graph of Q_g. It is well-known that f is strictly convex if and only if its Hessian is positive definite.

Notation for defining the Hessian of f: We use Ilkilic's (2010) notation for labeling links in a bipartite graph. We define firm j's supply to the domestic market as q_{jj}.

Let $\rho : L \leftarrow \mathbb{N}_+$ be a lexicographic order on L respecting ι[73] such that ρ relabels the (i,j) pairs from 1 to $r(g)$ by skipping those links which are not in g. The function ρ satisfies the following conditions:

[73]We will order all possible links such that the links of a firm f_j are assigned a lower number than any firm f_i for $i > j$, and the links of a firm are ordered according to the indices of the markets they are connected. The label of a possible link (i,j) will be denoted by $\iota(i,j)$.

1. $\exists (i,j) \in L$ such that $\rho(i,j) = 1$,

2. $(i,j) \neq (k,l) \implies \rho(i,j) \neq \rho(k,l)$,

3. $j \leq l \implies \rho(i,j) \leq \rho(k,l)$ for all (i,j), $(k,l) \in L$,

4. $i \leq k \implies \rho(i,j) \leq \rho(k,j)$ for all (i,j), $(k,l) \in L$,

5. If $\exists (i,j)$ s.t. $\rho(i,j) = z \geq 1$ then $\exists (k,l) \in L$ s.t. $\rho(k,l) = z - 1$.

Given a network g and for $t = \xi(i,j)$, the Hessian matrix of the potential function, $H = [g_{tz}]_{r(g) \times r(g)}$ is

$$g_{tz} = \begin{cases} 2\beta_i, & \text{if } t = z = \xi(i,j) \text{ for some } m_i \in M\,,\, f_j \in F \\ \beta_i, & \text{if } t \neq z\,,\, t = \xi(i,j)\,,\, z = \xi(i,k) \text{ for some } m_i \in M\,,\, f_j\,, f_k \in F \\ 0, & \text{otherwise.} \end{cases}$$

Next, we show that for any matrix H we can find a matrix R with independent columns such that $H = R^T R$, which is equivalent to checking that H is positive definite.

Create a diagonal matrix, $\dot{H}$, of size $r(g) \times r(g)$ with diagonal elements equal to square root of $\dfrac{1}{2} \times$ the corresponding diagonal element of H for those rows that have non-zero non-diagonal elements and diagonal elements equal to $2\times$ the corresponding diagonal element of H for those rows that have only zero non-diagonal elements.

Then, create a matrix, $\tilde{H}$, of size equal to (number of non-zero non-diagonal elements in the upper triangular of H divided by 2)$\times r(g)$[74]. Row l of $\tilde{H}$ is filled by:

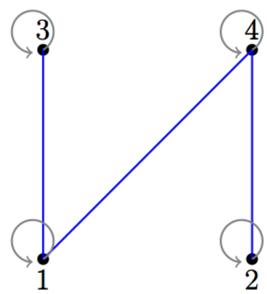
$$g_{lk} = \begin{cases} \sqrt{\beta_i}, & \text{for all columns } k = 1, 2, ..., r(g) \text{ where} \rho^{-1}(k) = q_{ij} \text{ if they share market } m_i \\ 0, & \text{otherwise.} \end{cases}$$

Apply this to all rows l in $\tilde{H}$. The final step involves combining the matrices $\dot{H}$ and $\tilde{H}$ by their rows to obtain a matrix R with a size of (number of non-zero

[74] Ignore the rows of H that have zero non-diagonal elements.

non-diagonal elements in the upper triangular of H divided by 2)$+\ r(g) \times r(g)$. One can easily show that $H = R^T R$ which completes the proof. Finally, the constraints in this problem are linear, which satisfies the requirement in *Theorem 2.3* in Zhu (2008) that they have to be convex.

Example: Here is the Hessian, H and the matrix, R in the simple example network provided in the chapter.



We write the vector of quantities supplied in this graph as:

$$Q_g = \begin{bmatrix} q_{11} & q_{31} & q_{41} & q_{22} & q_{42} & q_{33} & q_{44} \end{bmatrix}$$

$$H = \begin{bmatrix} 2\beta_1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 2\beta_3 & 0 & 0 & 0 & \beta_3 & 0 \\ 0 & 0 & 2\beta_4 & 0 & \beta_4 & 0 & \beta_4 \\ 0 & 0 & 0 & 2\beta_2 & 0 & 0 & 0 \\ 0 & 0 & \beta_4 & 0 & 2\beta_4 & 0 & \beta_4 \\ 0 & \beta_3 & 0 & 0 & 0 & 2\beta_3 & 0 \\ 0 & 0 & \beta_4 & 0 & \beta_4 & 0 & 2\beta_4 \end{bmatrix}$$

and

$$\dot{H} = \begin{bmatrix} \sqrt{2\beta_1} & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & \sqrt{\beta_3} & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \sqrt{\beta_4} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \sqrt{2\beta_2} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \sqrt{\beta_4} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & \sqrt{\beta_3} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & \sqrt{\beta_4} \end{bmatrix}$$

$$\tilde{H} = \begin{bmatrix} 0 & \sqrt{\beta_3} & 0 & 0 & 0 & \sqrt{\beta_3} & 0 \\ 0 & 0 & \sqrt{\beta_4} & 0 & \sqrt{\beta_4} & 0 & \sqrt{\beta_4} \end{bmatrix}$$

We combine the matrices $\dot{H}$ and $\tilde{H}$ by their rows to obtain:

$$R = \begin{bmatrix} \sqrt{2\beta_1} & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & \sqrt{\beta_3} & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \sqrt{\beta_4} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \sqrt{2\beta_2} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \sqrt{\beta_4} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & \sqrt{\beta_3} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & \sqrt{\beta_4} \\ 0 & \sqrt{\beta_3} & 0 & 0 & 0 & \sqrt{\beta_3} & 0 \\ 0 & 0 & \sqrt{\beta_4} & 0 & \sqrt{\beta_4} & 0 & \sqrt{\beta_4} \end{bmatrix}$$

1.8.2 First order conditions for calibration

The first order conditions that are used in calibration:

Europe:

$$\alpha_1 - 2\beta_1 q_{11} - \beta_1(q_{14} + q_{15} + q_{16} + q_{17} + q_{18}) - \lambda_1 - \mu_{11} = 0$$

North America:

$$\alpha_2 - 2\beta_2 q_{22} - \beta_2(q_{24} + q_{25} + q_{26}) - \lambda_2 - \mu_{22} = 0$$

Asia Pacific:

$$\alpha_3 - 2\beta_3 q_{33} - \beta_3(q_{35} + q_{37} + q_{38} + q_{39}) - \lambda_3 - \mu_{33} = 0$$

West Africa:

$$\alpha_1 - 2\beta_1 q_{15} - \beta_1(q_{11} + q_{14} + q_{16} + q_{17} + q_{18}) - \tau_{15} - \lambda_5 - \mu_{15} = 0$$

$$\alpha_2 - 2\beta_2 q_{25} - \beta_2(q_{22} + q_{24} + q_{26}) - \tau_{25} - \lambda_5 - \mu_{25} = 0$$

$$\alpha_3 - 2\beta_3 q_{35} - \beta_3(q_{33} + q_{37} + q_{38} + q_{39}) - \tau_{35} - \lambda_5 - \mu_{35} = 0$$

$$\alpha_5 - 2\beta_5 q_{55} - \lambda_5 - \mu_{55} = 0$$

North Africa:

$$\alpha_1 - 2\beta_1 q_{16} - \beta_1(q_{11} + q_{14} + q_{15} + q_{17} + q_{18}) - \tau_{16} - \lambda_6 - \mu_{16} = 0$$

$$\alpha_2 - 2\beta_2 q_{26} - \beta_2(q_{22} + q_{24} + q_{25}) - \tau_{26} - \lambda_6 - \mu_{26} = 0$$

$$\alpha_6 - 2\beta_6 q_{66} - \lambda_6 - \mu_{66} = 0$$

Russia:

$$\alpha_1 - 2\beta_1 q_{17} - \beta_1(q_{11} + q_{14} + q_{15} + q_{16} + q_{18}) - \tau_{17} - \lambda_7 - \mu_{17} = 0$$

$$\alpha_3 - 2\beta_3 q_{37} - \beta_3(q_{33} + q_{35} + q_{38} + q_{39}) - \tau_{37} - \lambda_7 - \mu_{37} = 0$$

$$\alpha_7 - 2\beta_7 q_{77} - \lambda_7 - \mu_{77} = 0$$

Middle East:

$$\alpha_1 - 2\beta_1 q_{18} - \beta_1(q_{11} + q_{14} + q_{15} + q_{16} + q_{18}) - \tau_{18} - \lambda_8 - \mu_{18} = 0$$

$$\alpha_3 - 2\beta_3 q_{38} - \beta_3(q_{33} + q_{35} + q_{37} + q_{39}) - \tau_{38} - \lambda_8 - \mu_{38} = 0$$

$$\alpha_8 - 2\beta_8 q_{88} - \lambda_8 - \mu_{88} = 0$$

Australasia:

$$\alpha_3 - 2\beta_3 q_{39} - \beta_3(q_{33} + q_{35} + q_{37} + q_{38}) - \tau_{39} - \lambda_9 - \mu_{39} = 0$$

$$\alpha_9 - 2\beta_9 q_{99} - \lambda_9 - \mu_{99} = 0$$

Table 1.1 : Price in each market

Price in Europe:	$\hat{p}_1 = \alpha_1 - \beta_1(q_{11} + q_{14} + q_{15} + q_{16} + q_{17} + q_{18})$
Price in North America:	$\hat{p}_2 = \alpha_2 - \beta_2(q_{22} + q_{24} + q_{25} + q_{26})$
Price in Asia Pacific:	$\hat{p}_3 = \alpha_3 - \beta_3(q_{33} + q_{35} + q_{37} + q_{38} + q_{39})$
Price in South America:	$\hat{p}_4 = \alpha_4 - \beta_4 q_{44}$
Price in West Africa:	$\hat{p}_5 = \alpha_5 - \beta_5 q_{55}$
Price in North Africa:	$\hat{p}_6 = \alpha_6 - \beta_6 q_{66}$
Price in Russia:	$\hat{p}_7 = \alpha_7 - \beta_7 q_{77}$
Price in Middle East:	$\hat{p}_8 = \alpha_8 - \beta_8 q_{88}$
Price in Australasia:	$\hat{p}_9 = \alpha_9 - \beta_9 q_{99}$

Table 1.2 : Network parameters

	Parameter	Value
Choke price in Europe:	α_1	904.27
Choke price in North America:	α_2	302.9
Choke price in Asia Pacific:	α_3	832.83
Choke price in South America:	α_4	260.02
Choke price in West Africa:	α_5	220.01
Choke price in North Africa:	α_6	199.97
Choke price in Russia:	α_7	130.03
Choke price in Middle East:	α_8	200.01
Choke price in Australasia:	α_9	239.99
Slope of European inverse demand curve:	β_1	1.041
Slope of North America's inverse demand curve:	β_2	0.184
Slope of Asia Pacific's inverse demand curve:	β_3	1.3003
Slope of South America's inverse demand curve:	β_4	0.965
Slope of West Africa's inverse demand curve:	β_5	10.912
Slope of North Africa's inverse demand curve:	β_6	1.445
Slope of Russian inverse demand curve:	β_7	0.134
Slope of Middle East's inverse demand curve:	β_8	0.2894
Slope of Australasian inverse demand curve:	β_9	1.083
Marginal cost of exporting from South America to Europe:	τ_{14}	292.08
Marginal cost of exporting from South America to North America:	τ_{24}	148.59
Marginal cost of exporting from West Africa to Europe:	τ_{15}	288.85
Marginal cost of exporting from West Africa to North America:	τ_{25}	149.43
Marginal cost of exporting from West Africa to Asia Pacific:	τ_{35}	311.41
Marginal cost of exporting from North Africa to Europe:	τ_{16}	230.02
Marginal cost of exporting from North Africa to North America:	τ_{26}	149.07
Marginal cost of exporting from Russia to Europe:	τ_{17}	111.41
Marginal cost of exporting from Russia to Asia Pacific:	τ_{37}	311.93
Marginal cost of exporting from Middle East to Europe:	τ_{18}	273.34
Marginal cost of exporting from Middle East to Asia Pacific:	τ_{38}	258.62
Marginal cost of exporting from Australasia to Asia Pacific:	τ_{39}	205.18

Table 1.3 : Equilibrium trade flows (in Bcm)

Route	2009	Scenario I	Scenario II	Scenario III	Scenario IV.a	Scenario IV.b	Scenario V	Scenario VI
From Europe to Europe	288.1	284.31	288.10	288.10	288.10	288.10	288.10	285.15
From North America to North America	813	813.00	813.00	813.00	813.00	833.61	813.00	813.00
From Asia Pacific to Asia Pacific	246.1	246.10	246.10	246.10	246.10	246.10	237.34	238.36
From South America to Europe	7.6	3.81	12.04	8.54	8.82	10.81	8.82	4.64
From South America to North America	7.6	7.63	4.41	7.11	9.22	5.06	6.58	7.64
From South America to South America	134.7	134.70	133.45	134.25	131.87	134.02	134.50	134.70
From West Africa to Europe	10.7	6.91	14.05	8.35	11.49	12.16	12.30	7.74
From West Africa to North America	3.1	3.09	0.00	0.00	2.23	0.00	4.15	3.08
From West Africa to Asia Pacific	6.6	8.32	6.51	12.25	6.95	8.38	0.00	0.00
From West Africa to West Africa	10.08	10.08	9.92	9.88	9.81	9.94	10.08	10.08
From North Africa to Europe	67.2	63.41	71.59	68.12	68.29	70.40	68.42	64.24
From North Africa to North America	5.0	5.00	1.46	4.38	5.85	2.26	3.92	5.00
From North Africa to North Africa	69.2	69.20	68.35	68.89	67.26	68.74	69.06	69.20
From Russia to Europe	181.1	177.31	187.76	181.00	185.58	184.06	173.87	169.94
From Russia to Asia Pacific	6.2	7.92	0.00	13.66	9.51	9.18	47.83	49.35
From Russia to Russia	485.5	485.44	485.04	478.14	477.70	479.56	451.10	453.52
From Middle East to Europe	25.6	48.32	0.00	24.41	28.93	27.68	27.20	51.55
From Middle East to Asia Pacific	47.2	41.41	49.85	53.79	49.59	49.48	38.44	33.87
From Middle East to Middle East	345.6	328.67	345.54	340.20	339.88	341.24	345.54	332.98
From Australasia to Asia Pacific	88.3	88.94	89.29	91.66	90.14	89.87	79.54	80.56
From Australasia to Australasia	110.8	110.16	109.81	107.44	108.96	109.23	110.80	110.80

Table 1.4 : Percentage change in equilibrium trade flows compared to 2009

Route	Scenario I	Scenario II	Scenario III	Scenario IV.a	Scenario IV.b	Scenario V	Scenario VI
From Europe to Europe	-1.31	0.00	0.00	0.00	0.00	0.00	-1.03
From North America to North America	0.00	0.00	0.00	0.00	2.54	0.00	0.00
From Asia Pacific to Asia Pacific	0.00	0.00	0.00	0.00	0.00	-3.56	-3.15
From South America to Europe	-49.86	58.46	12.35	16.00	42.28	16.08	-38.91
From South America to North America	0.40	-42.03	-6.41	21.29	-33.39	-13.42	0.48
From South America to South America	0.00	-0.93	-0.34	-2.10	-0.50	-0.15	0.00
From West Africa to Europe	-35.38	31.30	-22.01	7.34	13.64	14.93	-27.63
From West Africa to North America	-0.33	-100.00	-100.00	-27.99	-100.00	33.92	-0.64
From West Africa to Asia Pacific	26.00	-1.30	85.64	5.36	27.02	-100.00	-100.00
From West Africa to West Africa	0.00	-1.62	-1.96	-2.69	-1.42	0.00	0.00
From North Africa to Europe	-5.63	6.53	1.38	1.62	4.75	1.81	-4.40
From North Africa to North America	-0.01	-70.71	-12.25	17.12	-54.70	-21.58	-0.02
From North Africa to North Africa	0.00	-1.23	-0.44	-2.80	-0.66	-0.20	0.00
From Russia to Europe	-2.09	3.68	-0.05	2.48	1.63	-3.99	-6.17
From Russia to Asia Pacific	27.69	-100.00	120.33	53.48	48.11	671.56	696.02
From Russia to Russia	-0.01	-0.09	-1.52	-1.61	-1.22	-7.09	-6.59
From Middle East to Europe	88.76	-100.00	-4.64	12.99	8.12	6.24	101.36
From Middle East to Asia Pacific	-12.27	5.61	13.95	5.06	4.83	-18.56	-28.24
From Middle East to Middle East	-4.90	-0.02	-1.56	-1.65	-1.26	-0.02	-3.65
From Australasia to Asia Pacific	0.73	1.12	3.81	2.08	1.78	-9.92	-8.77
From Australasia to Australasia	-0.58	-0.90	-3.04	-1.66	-1.42	0.00	0.00

Table 1.5 : Producers' percentage market share in each importing region

Europe	2009	Scenario I	Scenario II	Scenario III	Scenario IV.a	Scenario IV.b	Scenario V	Scenario VI
Share of Europe	49.6	48.7	50.2	49.8	48.7	48.6	49.8	48.9
Share of South America	1.3	0.7	2.1	1.5	1.5	1.8	1.5	0.8
Share of West Africa	1.8	1.2	2.4	1.4	1.9	2.0	2.1	1.3
Share of North Africa	11.6	10.9	12.5	11.8	11.6	11.9	11.8	11.0
Share of Russia	31.2	30.4	32.7	31.3	31.4	31.0	30.0	29.1
Share of Middle East	4.4	8.3	0.0	4.2	4.9	4.7	4.7	8.8
North America	**2009**	**Scenario I**	**Scenario II**	**Scenario III**	**Scenario IV.a**	**Scenario IV.b**	**Scenario V**	**Scenario VI**
Share of North America	98.1	98.1	99.3	98.6	97.9	99.1	98.2	98.1
Share of South America	0.9	0.9	0.5	0.9	1.1	0.6	0.8	0.9
Share of West Africa	0.4	0.4	0.0	0.0	0.3	0.0	0.5	0.4
Share of North Africa	0.6	0.6	0.2	0.5	0.7	0.3	0.5	0.6
Asia Pacific	**2009**	**Scenario I**	**Scenario II**	**Scenario III**	**Scenario IV.a**	**Scenario IV.b**	**Scenario V**	**Scenario VI**
Share of Asia Pacific	62.4	62.7	62.8	59.0	61.2	61.1	58.9	59.3
Share of West Africa	1.7	2.1	1.7	2.9	1.7	2.1	0.0	0.0
Share of Russia	1.6	2.0	0.0	3.3	2.4	2.3	11.9	12.3
Share of Middle East	12.0	10.5	12.7	12.9	12.3	12.3	9.5	8.4
Share of Australasia	22.4	22.6	22.8	22.0	22.4	22.3	19.7	20.0

Table 1.6 : Percentage share of each market in exporters' total production

South America	2009	Scenario I	Scenario II	Scenario III	Scenario IV.a	Scenario IV.b	Scenario V	Scenario VI
Europe	5.1	2.6	8.0	5.7	5.9	7.2	5.9	3.2
North America	5.1	5.2	2.9	4.7	6.1	3.4	4.4	5.2
South America	89.9	92.2	89.0	89.6	88.0	89.4	89.7	91.6
West Africa	**2009**	**Scenario I**	**Scenario II**	**Scenario III**	**Scenario IV.a**	**Scenario IV.b**	**Scenario V**	**Scenario VI**
Europe	35.1	24.3	46.1	27.4	37.7	39.9	46.4	37.0
North America	10.2	10.9	0.0	0.0	7.3	0.0	15.6	14.7
Asia Pacific	21.7	29.3	21.4	40.2	22.8	27.5	0.0	0.0
West Africa	33.1	35.5	32.5	32.4	32.2	32.6	38.0	48.2
North Africa	**2009**	**Scenario I**	**Scenario II**	**Scenario III**	**Scenario IV.a**	**Scenario IV.b**	**Scenario V**	**Scenario VI**
Europe	47.5	46.1	50.6	48.2	48.3	49.8	48.4	46.4
North America	3.5	3.6	1.0	3.1	4.1	1.6	2.8	3.6
North Africa	48.9	50.3	48.3	48.7	47.6	48.6	48.8	50.0
Russia	**2009**	**Scenario I**	**Scenario II**	**Scenario III**	**Scenario IV.a**	**Scenario IV.b**	**Scenario V**	**Scenario VI**
Europe	26.9	26.4	27.9	26.9	27.6	27.4	25.8	25.3
Asia Pacific	0.9	1.2	0.0	2.0	1.4	1.4	7.1	7.3
Russia	72.2	72.4	72.1	71.1	71.0	71.3	67.0	67.4
Middle East	**2009**	**Scenario I**	**Scenario II**	**Scenario III**	**Scenario IV.a**	**Scenario IV.b**	**Scenario V**	**Scenario VI**
Europe	6.1	11.5	0.0	5.8	6.9	6.6	6.6	12.3
Asia Pacific	11.3	9.9	12.6	12.9	11.9	11.8	9.3	8.1
Middle East	82.6	78.6	87.4	81.3	81.2	81.6	84.0	79.6
Australasia	**2009**	**Scenario I**	**Scenario II**	**Scenario III**	**Scenario IV.a**	**Scenario IV.b**	**Scenario V**	**Scenario VI**
Asia Pacific	44.3	44.7	44.8	46.0	45.3	45.1	41.8	42.1
Australasia	55.7	55.3	55.2	54.0	54.7	54.9	58.2	57.9

Chapter 2

Effects of North American shale gas on world natural gas markets

2.1 Introduction

Recent developments in hydraulic fracturing and horizontal drilling have changed the course of natural gas trade for the U.S. Until a decade ago, market players were investing in LNG import facilities in the U.S., based on the predictions that U.S. domestic supply was becoming scarce. Contrary to these predictions, recent technological developments have unlocked natural gas resources and enabled a rapid growth of natural gas production in the U.S. For instance, according to the Energy Information Administration (EIA), gross withdrawals from shale gas wells in the U.S. has increased from zero in 2000 to over 23 billion cubic feet per day (Bcfd) in 2011, representing over 29 percent of total gross production in the U.S. These abundant supplies and the consequent low prices in the U.S. relative to the European and Asia Pacific markets have led natural gas producers to look at the profit opportunities from exporting to these markets. This chapter focuses on the possible impacts on domestic and international gas prices of U.S. natural gas exports.

As stated in the first chapter, most world natural gas production is concentrated in a small number of producers. However, following the Natural Gas Policy Act in 1978, the U.S. has developed the most liberalized natural gas market with a large number of producers of whom each has easy access to a huge market. In particular, ownership of transportation capacity rights is unbundled from pipeline ownership. Unbundling of capacity rights from facility ownership makes it possible for any producer to access markets through a competitive bid. By contrast, in most other markets globally, pipeline capacity is not unbundled from facility ownership, meaning large incumbent monopolies can effectively present barriers to entry through control of the transportation infrastructure.[1] We therefore consider a mixed oligopoly competitive model[2] of the world natural gas market, where we allow for OPEC type[3] of competition where there are dominant producers facing a fringe of small competitors, albeit largely confined to North America. In reality, small firms are also part of the market in some overseas jurisdictions, especially Australia, but for simplicity we assume the competitive fringe is present only in North America.

We model the world natural gas market in a network structure where buyers and sellers are connected through a trading network. We base our model on Ilkilic (2010), who develops a bipartite network model for m markets and n firms in Cournot competition and analyzes how the structure of network that connects suppliers with producers affects the market outcome. In this chapter, we relax Ilkilic's (2010) assumption that each producer is a Cournot player and has no capacity constraint. To better represent the North American natural gas market, this chapter allows for perfect competition in that market. We show that our game can be represented as a potential game and with a unique equilibrium.

In this chapter, we add more links to the world natural gas trade network that we introduced in the first chapter, but we still assume that the network graph is fixed. More specifically, we allow for buyers and sellers to be connected if the seller has a liquefaction facility and the buyer has a regasification facility. For instance in 2009, there was no natural gas trade between Trinidad and Tobago and Japan[4] but we

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[1]See the section by Medlock in "2013 Policy Recommendations for the Obama Administration" by the James A. Baker III Institute for Public Policy, March 2013.

[2]The literature uses "mixed oligopoly" to refer to a market with public and private firms, where the public firms maximize social welfare and private firms maximize their profits. In this chapter, we introduce a new term "mixed oligopoly competitive" to define a market which has a mix of competitive, monopolistic, monopsonistic and oligopolistic elements.

[3]Our model differs from the most common model of OPEC, which assumes there is a single dominant producer that is OPEC. Unlike in the oil market, in the world natural gas market a few dominant producers compete with each other.

[4]In our aggregated regions, Trinidad and Tobago belongs to South America and Japan belongs

allow for a link that connects South America to the Asia Pacific. We do not assume any exogenous trade volumes and instead let our model solve for equilibrium trade volumes. As a result, in the equilibrium there might be links that carry zero flows but they are strategically redundant.[5]

The literature on North American natural gas exports is very recent. Therefore, there are few studies that we can compare with ours. In January 2012, the U.S. Energy Information Administration (EIA) published a study on the price impacts of U.S. LNG exports. Their analysis treated U.S. LNG exports as exogenous and did not consider potential interactions between the U.S. LNG exports and the world natural gas market. Similar to the EIA's report, a Deloitte Center for Energy Solutions' study titled "Made in America: The Economic Impact of LNG Exports from the United States" assumed a particular volume of LNG exports from the U.S. when assessing the domestic price impact. It also did not allow any interaction between domestic and international markets to influence the volume of trade.

Contrary to these studies, Medlock (2012) suggested that one should consider U.S. LNG exports in a global setting and model them in an international trade framework. According to Medlock (2012), key factors that determine the impact of U.S. LNG exports on domestic prices are the elasticity of domestic supply and demand, the elasticity of foreign supply and demand, the role of short-term capacity constraints, the cost of developing and utilizing export capacity and the exchange rate. Most of these factors were ignored in the previous studies. His analysis, using the Rice World Gas Trade Model (RWGTM), concluded that the natural gas spot price differentials between Asia and the U.S. and Europe and the U.S. are not sufficient to support long-term LNG exports from the U.S. Gulf Coast to these regions.

Shortly after Medlock (2012), in December 2012, the U.S. Department of En-

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to the Asia Pacific.

[5]Theorem 2 of Ilkilic (2010) tells us that the links which carry no flows in the equilibrium are strategically redundant; they play no role in determining the equilibrium. For the firms of such links, the marginal profits of supplying via them are not positive. They are indifferent between having such a link or not.

ergy has published another study[6] titled "Macroeconomic Impacts of LNG Exports from the United States" which assesses the potential macroeconomic impacts of LNG exports. Compared to the previous studies, the DOE study considered the world natural gas market in a global setting but failed to model it realistically. Specifically, they assumed that the world natural gas market had a single dominant supplier, with the largest share of LNG exports. The dominant supplier was assumed to limit output, but had to contend with competitive fringe whose production adjusts to market prices. Their calculations of U.S. benefits from LNG exports assumed that the dominant supplier would not change its plan for expanding production to counter U.S. entry to market. Therefore, they concluded that there will be demand for U.S. LNG exports.

Like the EIA (2012) paper and unlike the RWGTM, we assume that the world natural gas market is imperfectly competitive. However, we allow all production, consumption, trade and pricing outcomes to be endogenous. In particular, to the best of our knowledge, this is the first explicit non-competitive model, where export volumes from North America and their price impacts are endogenous.[7]

We find that North American LNG exports occur when its elasticity of supply is very high and Asian demand remains strong.[8,9] The volume of natural gas that North America exports decrease as its competitors' elasticities of supply increase. We also find that a more elastic North American supply curve reduces natural gas producers' market power all around the world.

In Section 2.2, we introduce our schematic representation of the world natural

[6]This study was done by NERA Economic Consulting at the request of the U.S. Department of Energy, Office of Fossil Energy.

[7]The RWGTM has endogenous export volumes and prices, but it is competitive.

[8]More explicitly, we show that North America exports LNG when the North American supply curve is at least 75 percent flatter than its original supply curve in 2009 and Asia Pacific's demand is increased by at least 15 percent relative to 2009.

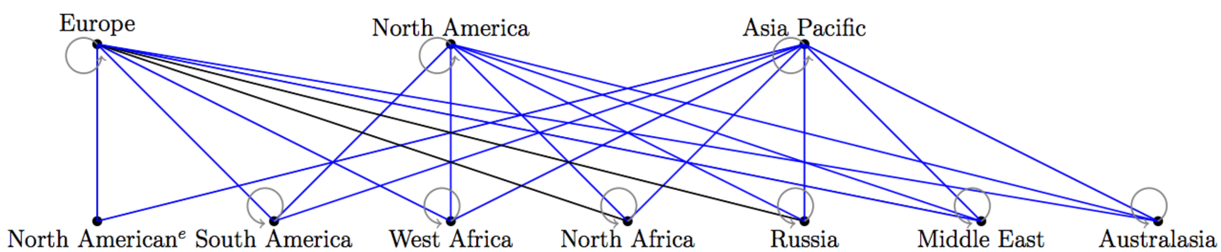
[9]An unrealistic feature of our model is that North America does not export and import at the same time. This is because we assume a single price for the North American market. However, in reality due to pipeline capacity constraints it can export LNG from the Gulf Coast and import to New England.

gas trade network. In Section 2.3, we introduce our fringe and dominant producers' Cournot game and solve for its unique Cournot-Nash equilibrium. Section 2.4 is devoted to analyzing different policy scenarios. Section 2.5 concludes. In the appendix, we calibrate the model parameters by using natural gas production, consumption, trade, proved reserves and prices in 2009.

2.2 Schematic representation of the world natural gas market

As in the first chapter, we aggregate producers and consumers into a small number of regions. Below is the schematic representation of the world natural gas network used in this chapter.[10] We denote the North American exporter[11] by North American^e.

Figure 2.1 : Schematic representation



2.3 Model

The notation of this chapter is same as the first chapter.

2.3.1 Fringe and dominant producers' Cournot game

Given a graph g, each firm f_j maximizes its profit by supplying non-negative quantities to the markets in $N_g(f_j)$. Thus, the set of strategic players is the set of firms F. Let

[10] The blue lines indicate that the natural gas is transported via LNG and the black lines indicate that the natural gas is transported via pipeline. Half of the natural gas exports from North Africa to Europe are carried via LNG and half of them are carried via pipeline.

[11] It purchases natural gas at domestic prices and exports to the Asia Pacific and Europe.

$q_{ij} \geq 0$ be the quantity supplied by firm f_j to market d_i and Ω_g be the vector of quantities supplied in graph g. Let $r(g)$ be the size of Ω_g and assume we list the supply q_{ij} above the supply q_{kl} when $j < l$ or $j = l$ and $i < k$.

We assume that markets have linear inverse demand functions. Specifically, given a market d_i and a flow vector Ω_g the price, p_i, at d_i is

$$p_i(\Omega_g) = \alpha_i - \beta_i h_i, \tag{2.1}$$

where α_i and β_i are positive constants and h_i is natural gas consumption in market d_i:

$$h_i = \sum_{f_j \in N_g(d_i)} q_{ij}. \tag{2.2}$$

For example, the total consumption in North America in our trade network is $h_2 = q_{22} + q_{24} + q_{25} + q_{26} + q_{27} + q_{28} + q_{29} - q_{12} - q_{32}$ leading to linear inverse demand $p_2 = \alpha_2 - \beta_2(q_{22} + q_{24} + q_{25} + q_{26} + q_{27} + q_{28} + q_{29} - q_{12} - q_{32})$.

We assume that each natural gas producer has quadratic costs of production in the short run up to its production capacity, $\bar{S}_j$:

$$TC_j(\Omega_g) = \frac{\gamma_j}{2}\left[\sum_{d_i \in N_g(f_j)} q_{ij}\right]^2 \text{ where } \sum_{d_i \in N_g(f_j)} q_{ij} \leq \bar{S}_j, \tag{2.3}$$

We also assume that the cost of exporting natural gas is linear. For firm f_j the short-run total cost of export therefore is

$$T_j(\Omega_g) = \sum_{d_i \in N_g(f_j) \setminus d_j} \tau_{ij} q_{ij}, \tag{2.4}$$

where τ_{ij} is the marginal cost of exporting natural gas to market i.[12] If the natural gas is carried via LNG, τ_{ij} includes[13] the port-to-port cost of shipment, and the costs

[12] We assume that cost of exporting natural gas is proportional to the export volume.

[13] These costs are per unit of natural gas, that is one Bcm in this chapter.

of liquefaction and regasification. If the natural gas is carried via pipeline, τ_{ij} includes tariffs paid to transit countries, the cost of fuel lost during transportation, and the cost of operations and maintenance of the pipeline.

2.3.2 North American fringe producers

We assume that the domestic producers in North America form a competitive fringe. Each member of the fringe is too small to influence the market price, and takes the equilibrium price to be given independently of its own actions.

Let the aggregate cost function of competitive fringe firms in North America be

$$TC(\Omega_g) = \frac{\gamma_2}{2} Q_2^2 \tag{2.5}$$

where Q_2 is the total supply of fringe producers in North America, which is the sum of North American domestic demand and its exports to Europe and the Asia Pacific.[14]

In equilibrium, fringe producers supply at $p_2 = \gamma_2 \, Q_2 \implies Q_2^\star = \dfrac{p_2}{\gamma_2} \implies q_{22} = \dfrac{p_2}{\gamma_2} - q_{12} - q_{32}$.

A dominant producer's demand curve in each market is not the same as the market demand curve. Their demand curve is the difference between market demand and the supply from fringe firms.

The inverse demand function that dominant firms[15] face in North America is

$$p_2 = \alpha_2 - \beta_2 \left(\overbrace{\frac{p_2}{\gamma_2} - q_{12} - q_{32}}^{\text{Fringe's supply}} + \overbrace{q_{24} + q_{25} + q_{26} + q_{27} + q_{28} + q_{29}}^{\text{Dominant firms' supply}} \right). \tag{2.6a}$$

[14]That is,

$$Q_2 = \overbrace{q_{22}}^{\text{Domestic Demand}} + \overbrace{q_{12} + q_{32}}^{\text{Export Supply}}.$$

[15]The dominant firms that are connected to North America are South America, West Africa, North Africa, Russia, the Middle East and Australasia.

Re-arranging (2.6) we obtain:

$$p_2 \left(1 + \frac{\beta_2}{\gamma_2}\right) = \alpha_2 - \beta_2(q_{24} + q_{25} + q_{26} + q_{27} + q_{28} + q_{29} - q_{12} - q_{32}), \tag{2.6b}$$

which can then be solved for p_2:

$$p_2 = \underbrace{\left(\frac{\gamma_2}{\gamma_2 + \beta_2}\right)\alpha_2}_{=\tilde{\alpha}_2} - \underbrace{\left(\frac{\gamma_2}{\gamma_2 + \beta_2}\right)\beta_2}_{=\tilde{\beta}_2}(q_{24}+q_{25}+q_{26}+q_{27}+q_{28}+q_{29}-q_{12}-q_{32}). \tag{2.6c}$$

Since we assume that there is a single price for each region, North America does not export and import at the same time. Therefore, total exports to North America, $q_{24} + q_{25} + q_{26} + q_{27} + q_{28} + q_{29}$, and the total exports from North America, $q_{12} + q_{32}$, are not positive in the equilibrium of a same scenario.[16]

2.3.3 North America as an exporter

The North American exporter, when it exists, is a monopsony buyer who purchases natural gas from the domestic fringe producers at the price, represented in (2.6c) and ships to Europe and/or the Asia Pacific. The North American exporter then maximizes its profit by choosing q_{12} and q_{32}.

$$\begin{aligned} \max_{q_{12},\, q_{32} \geq 0} \{ & (\alpha_1 - \beta_1 h_1)\, q_{12} + (\alpha_3 - \beta_3 h_3)\, q_{32} - \tau_{12} q_{12} - \tau_{32} q_{32} \\ & - (\tilde{\alpha}_2 - \tilde{\beta}_2(q_{24} + q_{25} + q_{26} + q_{27} + q_{28} + q_{29} - q_{12} - q_{32}))\,(q_{12} + q_{32})\} \end{aligned} \tag{2.7}$$

where $h_1 = q_{11} + q_{12} + q_{14} + q_{15} + q_{16} + q_{17} + q_{18} + q_{19}$ and $h_3 = q_{32} + q_{33} + q_{34} + q_{35} + q_{36} + q_{37} + q_{38} + q_{39}$.

[16] To have a unique equilibrium, a bipartition of the network graph is required. If there was more than one price for North America, such as one price in New England and one price in Houston, then we would have to disaggregate North America into two regions: New England as an importing region and Houston as an exporting region. For now, we assume North America as a one region; either an exporting or an importing region.

The first order conditions are

$$
\begin{aligned}
q_{12}:\ \ & \alpha_1 - \beta_1\left(q_{11}+q_{14}+q_{15}+q_{16}+q_{17}+q_{18}+q_{19}\right) - 2\beta_1 q_{12} - \tau_{12} - \tilde{\alpha}_2 \\
& + \tilde{\beta}_2(q_{24}+q_{25}+q_{26}+q_{27}+q_{28}+q_{29}) - 2\tilde{\beta}_2(q_{12}+q_{32}) = 0
\end{aligned}
$$

$$
\begin{aligned}
q_{32}:\ \ & \alpha_3 - \beta_3\left(q_{33}+q_{34}+q_{35}+q_{36}+q_{37}+q_{38}+q_{39}\right) - 2\beta_3 q_{32} - \tau_{32} - \tilde{\alpha}_2 \\
& + \tilde{\beta}_2(q_{24}+q_{25}+q_{26}+q_{27}+q_{28}+q_{29}) - 2\tilde{\beta}_2(q_{12}+q_{32}) = 0
\end{aligned}
$$

Equilibrium exports to Europe are

$$
\begin{aligned}
q^{\star}_{12} =& \frac{\alpha_1 - \tilde{\alpha}_2 - \beta_1\left(q_{11}+q_{14}+q_{15}+q_{16}+q_{17}+q_{18}+q_{19}\right) + \tilde{\beta}_2(q_{24}+q_{25}+q_{26}+q_{27}+q_{28}+q_{29})}{2\beta_1 + 2\tilde{\beta}_2} \\
& - \frac{2\tilde{\beta}_2 q_{32} + \tau_{12}}{2\beta_1 + 2\tilde{\beta}_2} \qquad (2.8)
\end{aligned}
$$

while equilibrium exports to the Asia Pacific are

$$
\begin{aligned}
q^{\star}_{32} =& \frac{\alpha_3 - \tilde{\alpha}_2 - \beta_3\left(q_{33}+q_{34}+q_{35}+q_{36}+q_{37}+q_{38}+q_{39}\right) + \tilde{\beta}_2(q_{24}+q_{25}+q_{26}+q_{27}+q_{28}+q_{29})}{2\beta_3 + 2\tilde{\beta}_2} \\
& - \frac{2\tilde{\beta}_2 q_{12} + \tau_{32}}{2\beta_3 + 2\tilde{\beta}_2} \qquad (2.9)
\end{aligned}
$$

As we see in (2.8) and (2.9) LNG exports from North America depend on the elasticities of domestic supply and demand, the elasticities of foreign supply and demand,[17] and the role of short-run capacity constraints.[18]

---

[17]For instance, equilibrium supply of Europe to its domestic market, that is q_{11} depends on its elasticity of supply.

[18]Each dominant producer has a supply constraint in the short-run.

2.3.4 Dominant producers

In the world natural gas trade network, there are *eight* dominant producers[19] who have monopoly power in their domestic markets but compete with each other in the foreign market. Each dominant producer's objective is to maximize its profits subject to its supply constraint.

Example: West Africa's producer, labelled as 5, aims to:

$$\begin{aligned}\max_{q_{15},q_{25},q_{35},q_{55}} &\{(\alpha_1 - \beta_1(q_{11}+q_{12}+q_{14}+q_{15}+q_{16}+q_{17}+q_{18}+q_{19}))q_{15} \\ &+ (\tilde{\alpha}_2 - \tilde{\beta}_2(q_{24}+q_{25}+q_{26}+q_{27}+q_{28}+q_{29}-q_{12}-q_{32}))q_{25} \\ &+ (\alpha_3 - \beta_3(q_{32}+q_{33}+q_{34}+q_{35}+q_{37}+q_{38}+q_{39}))q_{35} \\ &+ (\alpha_5 - \beta_5 q_{55})q_{55} - \frac{\gamma_5}{2}(q_{15}+q_{25}+q_{35}+q_{55})^2 - \tau_{15}q_{15} - \tau_{25}q_{25} - \tau_{35}q_{35}\}\end{aligned} \tag{2.10}$$

subject to

$$q_{15}+q_{25}+q_{35}+q_{55} \leq \bar{S}_5 \quad \text{and} \quad q_{15}, q_{25}, q_{35}, q_{55} \geq 0. \tag{2.11}$$

We get the first order conditions as:

$$\begin{aligned} q_{15}:\ &\alpha_1 - 2\beta_1 q_{15} - \beta_1(q_{11}+q_{12}+q_{14}+q_{16}+q_{17}+q_{18}) - \gamma_5(q_{15}+q_{25}+q_{35}+q_{55}) \\ &- \tau_{15} - \lambda_5 - \mu_{15} = 0 \end{aligned}$$

$$\begin{aligned} q_{25}:\ &\tilde{\alpha}_2 - 2\tilde{\beta}_2 q_{25} - \tilde{\beta}_2(q_{24}+q_{26}+q_{26}+q_{27}+q_{28}+q_{29}-q_{12}-q_{32}) \\ &- \gamma_5(q_{15}+q_{25}+q_{35}+q_{55}) - \tau_{25} - \lambda_5 - \mu_{25} = 0 \end{aligned}$$

$$\begin{aligned} q_{35}:\ &\alpha_3 - 2\beta_3 q_{35} - \beta_3(q_{32}+q_{33}+q_{36}+q_{37}+q_{38}+q_{39}) - \gamma_5(q_{15}+q_{25}+q_{35}+q_{55}) \\ &- \tau_{35} - \lambda_5 - \mu_{35} = 0 \end{aligned}$$

[19]Unlike the rest of the producers, North America is a fringe producer. However, the North American exporter is a dominant supplier who competes with the dominant producers in the foreign markets.

$$q_{55}: \quad \alpha_5 - 2\beta_5 q_{55} - \gamma_5(q_{15} + q_{25} + q_{35} + q_{55}) - \lambda_5 - \mu_{55} = 0$$

where λ_5 is the Lagrange multiplier of the capacity constraint in (2.11), which can be interpreted as the shadow price of expanding capacity.

The stylized representation of the current world natural gas model that described above is a non-cooperative game with coupled payoff functions and coupled constraints.[20] Applying the Lagrangian multiplier theory to our model is not computationally convenient. This is because we have a large number of first-order conditions with inequality constraints (one for each producer) that need to be solved simultaneously. Instead, we use the potential game method introduced by Monderer and Shapley (1994).

Definition 1: Consider the Cournot game that we describe above with linear inverse demand functions, quadratic cost of productions and linear costs of exporting. Assume that there is one non-producing exporter, denoted as f_e, buying from one competitive domestic market, denoted as d_e. The exporter, f_e, supplies other importing markets which are imperfectly competitive. We denote the consumers in the exporter's domestic market as d_e. Then, we define a function

[20] The coupling arises because producers have a limited production capacity, which in turn limits their ability to supply markets to which they are connected.

$$
\begin{aligned}
P^{\star}(Q_g) = & \sum_{d_i \in N_g(f_j)\backslash d_e} \alpha_i \left(\sum_{f_j \in N_g(d_i)} q_{ij} \right) - \sum_{d_i \in N_g(f_j)\backslash d_e} \beta_i \left(\sum_{f_j \in N_g(d_i)} q_{ij}^2 \right) \\
& - \sum_{d_i \in N_g(f_j)\backslash d_e} \beta_i \left(\sum_{1 \leq j < k \leq n} q_{ij} q_{ik} \right) - \sum_{d_i \in N_g(f_j)\backslash d_j} \sum_{f_j \in N_g(d_i)} \tau_{ij} q_{ij} \\
& - \frac{\beta_e \gamma_e}{\gamma_e + \beta_e} \left(\sum_{1 \leq j < k \leq n} q_{ej} q_{ek} \right) + \frac{\alpha_e \gamma_e}{\gamma_e + \beta_e} \left(\sum_{f_j \in N_g(d_e)\backslash f_e} q_{ej} \right) \\
& - \frac{\beta_e \gamma_e}{\gamma_e + \beta_e} \left(\sum_{f_j \in N_g(d_e)\backslash f_e} q_{ej}^2 \right) + \frac{\beta_e \gamma_e}{\gamma_e + \beta_e} \left(\sum_{f_j \in N_g(d_e)\backslash f_e} q_{ej} \right) \left(\sum_{d_i \in N_g(f_e)\backslash d_e} q_{ie} \right) \\
& - \frac{\beta_e \gamma_e}{\gamma_e + \beta_e} \left(\sum_{d_i \in N_g(f_e)\backslash d_e} q_{ie} \right)^2 - \frac{\alpha_e \gamma_e}{\gamma_e + \beta_e} \sum_{d_i \in N_g(f_e)\backslash d_e} q_{ie} \\
& - \sum_{f_j \in N_g(d_i)\backslash f_e} \frac{\gamma_j}{2} \left(\sum_{d_i \in N_g(f_j)} q_{ij} \right)^2
\end{aligned}
\tag{2.12}
$$

subject to

$$\bar{S}_j \geq \sum_{d_i \in N_g(f_j)} q_{ij} \quad \text{for all} \;\; j \in F \tag{2.13}$$

and

$$q_{ij} \geq 0 \quad \text{for all} \;\; (i,j) \in g \tag{2.14}$$

It can be verified that for every link from firm j to market i, that is q_{ij}, and for every link that is not from firm j to market i, that is q_{-ij}, $P^{\star}(Q_g)$[21] satisfies[22]

$$\pi_j(q_{ij}, q_{-ij}) - \pi_j(x_{ij}, q_{-ij}) = P^{\star}(q_{ij}, q_{-ij}) - P^{\star}(x_{ij}, q_{-ij}) \tag{2.15}$$

and

[21] Q_g is the vector of quantities in graph g.

[22] π_j is the optimization problem of dominant firm j in the noncooperative game with constraints.

$$\pi_e(q_{ie}, q_{-ij}) - \pi_e(x_{ie}, q_{-ij}) = P^\star(q_{ie}, q_{-ij}) - P^\star(x_{ie}, q_{-ij}) \tag{2.16}$$

A function $P^\star$ satisfying (2.15) and (2.16) is called a potential function which requires

$$\frac{\partial \pi_j}{\partial q_{ij}} = \frac{\partial P^\star}{\partial q_{ij}} \quad \text{for all} \ \ (i,j) \in g \tag{2.17}$$

$$\frac{\partial \pi_e}{\partial q_{ie}} = \frac{\partial P^\star}{\partial q_{ie}} \quad \text{for all} \ \ (i,e) \in g \tag{2.18}$$

Theorem 1: The solution to the potential game defined in (2.12) subject to constraints defined in (2.13) and (2.14) is unique.

$$\max_{q_{ij}} P^\star(Q_g) \quad \text{for all} \ \ (i,j) \in g \tag{2.19}$$

More specifically, dominant firm j, f_j, maximizes

$$\begin{aligned}\pi_j(q_{ij}) = &\sum_{d_i \in N_g(f_j)\backslash d_e} \alpha_i q_{ij} - \sum_{d_i \in N_g(f_j)\backslash d_e} \beta_i q_{ij}^2 - \sum_{d_i \in N_g(f_j)\backslash d_e} \beta_i q_{ij} \left(\sum_{f_k \in N_g(d_i)\backslash f_j} q_{ik} \right) - \frac{\gamma_j}{2} \left(\sum_{f_j \in N_g(d_i)} q_{ij} \right)^2 \\ &- \sum_{f_j \in N_g(d_i)} \tau_{ij} q_{ij} + \sum_{d_e \in N_g(f_j)} \frac{\alpha_e \gamma_e}{\gamma_e + \beta_e} q_{ej} - \sum_{d_e \in N_g(f_j)} \frac{\beta_e \gamma_e}{\gamma_e + \beta_e} q_{ej}^2 - \sum_{1 \le j < k \le n} \frac{\beta_e \gamma_e}{\gamma_e + \beta_e} q_{ej} q_{ek} \\ &+ \sum_{d_i \in N_g(f_e)\backslash d_e} \frac{\beta_e \gamma_e}{\gamma_e + \beta_e} q_{ej} q_{ie}\end{aligned}$$

and the exporter e, f_e, maximizes

$$\begin{aligned}\pi_e(q_{ie}) = &\sum_{d_i \in N_g(f_e)\backslash d_e} \alpha_i q_{ie} - \sum_{d_i \in N_g(f_e)\backslash d_e} \beta_i q_{ie}^2 - \sum_{d_i \in N_g(f_e)\backslash d_e} \beta_i q_{ie} \left(\sum_{f_k \in N_g(d_i)\backslash f_e} q_{ik} \right) \\ &- \sum_{d_i \in N_g(f_e)\backslash d_e} \frac{\alpha_e \gamma_e}{\gamma_e + \beta_e} q_{ie} - \frac{\beta_e \gamma_e}{\gamma_e + \beta_e} \left(\sum_{d_i \in N_g(f_e)\backslash d_e} q_{ie} \right)^2 + \frac{\beta_e \gamma_e}{\gamma_e + \beta_e} \left(\sum_{f_j \in N_g(d_e)\backslash f_e} q_{ej} \right) \left(\sum_{d_i \in N_g(f_e)\backslash d_e} q_{ie} \right) \\ &- \sum_{d_i \in N_g(f_e)\backslash d_e} \tau_{ie} q_{ie}\end{aligned}$$

subject to their supply capacity constraint (of dominant producer) and non-negativity constraints.

subject to (2.13) and (2.14).

Proof: See Section (2.6.1).

2.4 Scenario analysis

In this section we analyze various scenarios[23] in the world natural gas market by changing the model's parameters[24] exogenously. With each of these changes we optimize a pair of new potential functions subject to new set of constraints.[25] Then, we compare our results with our reference case, that is world natural gas trade in 2009.

2.4.1 Scenario I: Change in North America's elasticity of domestic supply

Our motivation in this chapter is to understand the impact of shale gas developments on the world natural gas market. The elasticity of domestic supply is a critical determinant of the domestic price changes resulting from increased exports. We therefore change the slope of North American supply curve while holding all else constant.

In our first experiment, we decreased the slope of North American supply curve by 40 percent. The price in North America decreases from 150 million USD per Bcm to 99.91 million USD per Bcm, and the consumption in North America increases from 828 Bcm to 902.61 Bcm, and its natural gas imports are zero. Dominant producers which exported to North America in 2009, namely South America, West Africa and North Africa, shift their supply to Europe, the Asia Pacific and their domestic markets. For instance, South America increases its supply to Europe from 7.6 Bcm to 9.2 Bcm, while North Africa increases its supply to Europe from 67.2 Bcm to 67.75 Bcm. As a result, there will be more competition in the European and Asia Pacific

[23] Equilibrium trade flows under each scenario are provided in Table (2.2), Table (2.3) and Table (2.4).

[24] For calibration of these parameters see the appendix (2.6.2).

[25] We use the sequential quadratic programming algorithm in MATLAB's optimization toolbox to solve for the constrained optimum.

markets for Russia, the Middle East and Australasia. In response, they decrease their their supply to Europe and the Asia Pacific. For example, Russia's supply to Europe decreases by 0.56 Bcm and its supply to the Asia Pacific decreases by 0.04 Bcm.

A 40 percent decrease in the slope of North American supply curve is not sufficient for the North American exporter to export natural gas to Europe and/or the Asia Pacific. If we allow for a perfectly elastic North American supply curve, however, then North America exports 19.71 Bcm of natural gas to Europe and 17.48 Bcm of natural gas to the Asia Pacific. With these exports, there will be more competition for all dominant producers in the European or Asia Pacific markets. To prevent a further decline in equilibrium prices, the dominant producers decrease supply to both markets. For instance, Russia decreases its equilibrium supply to Europe from 181.1 Bcm to 177.59 Bcm and to the Asia Pacific from 6.2 Bcm to 3.13 Bcm. Under this scenario, equilibrium total supply to Europe increases from 580.3 Bcm to 584.6 Bcm, which decreases the equilibrium price from 300 million USD per Bcm to 296.98 million USD per Bcm. Similarly, equilibrium total supply to the Asia Pacific increases from 394.4 Bcm to 398 Bcm, which decreases the equilibrium price from 320 million USD per Bcm to 316.56 million USD per Bcm. The decline in equilibrium prices decrease the profits of dominant producers. For instance, the profit of the Asia Pacific declines by 1.4 billion USD. Similarly, the profit of Russia declines by 1 billion USD.

All else constant, North America starts exporting when the slope of its supply curve is 90 percent less than that of 2009. With a 90 percent lower slope, North America exports 0.7 Bcm to Europe and 0.83 Bcm to the Asia Pacific.

On the other hand, North America stops importing natural gas if we decrease the slope of North American supply curve by more than just 4 percent. With the 4 percent decline in the slope of its supply curve, it imports 0.47 Bcm of natural gas from South America. There is thus a big wedge in supply curve slopes where North American natural gas is a non-traded good, neither exported not imported.

Further analysis on Scenario I

In this section, first we shift North American supply curve[26] upward and keep its slope constant, and then decrease its slope.

If we increase the intercept of North American supply curve from zero to 19, then equilibrium exports to North America increase from 15.69 Bcm to 48.61 Bcm. An upward shift in the North American supply curve increases the marginal profits of all producers that are connected to it. This shift increases the equilibrium price in North America from by 10 million USD per Bcm.

Next, we assume a flat North American supply curve with a slope of 19. With this change, North America again becomes an exporter. It exports 0.6 Bcm to Europe and 0.75 Bcm to the Asia Pacific and all dominant producers shift their resources to Europe and the Asia Pacific. For instance, South America increases its supply to Europe from 7.6 Bcm to 9.17 Bcm while decreasing its supply to North America from 7.6 Bcm to zero.

2.4.2 Scenario II: An increase in Asia Pacific's natural gas demand

According to EIA projections, natural gas demand in Asia will grow from 341 Bcm to 497 Bcm from 2008 to 2015. We incorporate these projections into our model by increasing the choke price in the Asia Pacific by 15 percent. This shifts the demand curve out in a parallel fashion. We analyze the impact of this demand increase with a 40 percent, and a 75 percent decrease in the slope of North American supply curve, and with a flat North American supply curve.

If we allow for a natural gas demand increase in the Asia Pacific without changing the slope of North American supply curve, then total natural gas exports to North America decrease to 6.1 Bcm.[27] This occurs because all of the producers that are

[26]In our model, the North American supply curve is $p_2 = \gamma_2 Q_{22}$ however in this section we assume it to be $p_2 = \kappa_2 + \gamma_2 Q_{22}$ where we test for different values of κ_2 and/or γ_2.

[27]The link from South America carries 4.51 Bcm and the link from North Africa carries 1.59 Bcm of natural gas.

connected to the Asia Pacific respond to the demand increase by shifting supply from Europe, North America and their domestic markets to the Asia Pacific. For instance, West Africa increases its supply to the Asia Pacific from 6.6 Bcm to 21.23 Bcm by decreasing its supply to Europe from 10.7 Bcm to zero, and to North America from 3.1 Bcm to zero, and to its domestic market from 10.07 Bcm to 9.24 Bcm. The decline in total supply to the European market from West Africa, Russia and the Middle East will result in less competition in Europe for South America and North Africa. In response, South America and North Africa shift their resources from North America and their domestic markets to Europe.

Under this scenario, equilibrium total supply to Europe decreases by 7.1 Bcm, which increases the equilibrium price by 4.97 million USD per Bcm. Similarly, equilibrium total supply to North America decreases by 2.05 Bcm, which increases the equilibrium price by 1.39 million USD per Bcm. On the other hand, equilibrium supply to the Asia Pacific increases by 77.73 Bcm and the equilibrium price increases by 30.49 million USD per Bcm due to the upward shift in the Asia Pacific demand curve.

If, on top of the Asian demand increases, we decrease the slope of the North American supply curve by 40 percent, then total exports to North America decrease to zero. Under this scenario, North America does not export or import any natural gas.

If we decrease the slope of the North American supply curve by 75 percent, then North America exports 3.75 Bcm of natural gas to the Asia Pacific. The equilibrium prices in North America decrease from 150 million USD per Bcm to 45.48 million USD per Bcm. However, if we decrease the slope of North American supply curve by 75 percent but do not allow for exports, the equilibrium price decreases to 45.32 million USD per Bcm. Therefore, the impact of 3.75 Bcm of natural gas exports on North American domestic natural gas prices is 0.16 million USD per Bcm.

Next, we assume that North America has a flat supply curve. With a flat supply curve, North America exports 22.45 Bcm of natural gas to Europe and 39.97 Bcm of natural gas to the Asia Pacific. In the new equilibrium, total supply to the Asia Pacific increases by 91.41 Bcm, but the equilibrium price in the Asia Pacific also increases.[28] Profits of dominant producers decrease as the North American supply curve becomes more elastic.

Further analysis on Scenario II

In this section, first we increase the intercept of North American supply curve and keep its slope constant, and then decrease its slope.[29]

If we increase the intercept of North American supply curve to 5, then the equilibrium exports to North America decrease by 2.77 Bcm.[30] This results in an increase of 4.38 million USD per Bcm in North American prices. However, a 15 percent increase in the Asia Pacific choke price increase North American prices by only 1.39 million USD per Bcm.

Next, we assume a flat North American supply curve with an intercept of 50.[31] Then, North America becomes an exporter, and it exports 0.28 Bcm to the Asia Pacific. A flat North American supply curve with an intercept of 50 makes all producers shift supply from North America to Europe and the Asia Pacific. For instance, South America increases supply to Europe from 7.6 Bcm to 14.24 Bcm.

2.4.3 Scenario III: Decreased competition between Russia and the Middle East

In this scenario, we assume that Russia and the Middle East collude to maximize their joint profits. Our motivation is to understand the impact of shale gas developments in

[28]It increases by 17.65 million USD per Bcm. This is due to the upward shift in the Asia Pacific demand curve.

[29]Note that the choke price in the Asia Pacific is also increased by 15 percent in all these cases.

[30]Among the dominant producers, only South America increases its exports to North America.

[31]For any intercept that is higher than 50, North America does not export even though its supply curve is flat.

North America and resulting exports from it under such collusion. After the merger, the joint objective of Russia and the Middle East[32] is

$$
\begin{aligned}
\Pi_{78} &= \alpha_1(q_{17}+q_{18}) - \beta_1(q_{11}+q_{12}+q_{14}+q_{15}+q_{16}+q_{17}+q_{18}+q_{19})(q_{17}+q_{18}) \\
&+ \alpha_3(q_{37}+q_{38}) - \beta_3(q_{33}+q_{32}+q_{34}+q_{35}+q_{36}+q_{37}+q_{38}+q_{39})(q_{37}+q_{38}) \\
&+ \tilde{\alpha}_2(q_{27}+q_{28}) - \tilde{\beta}_2(q_{24}+q_{25}+q_{26}+q_{27}+q_{28}+q_{29}-q_{12}-q_{32})(q_{27}+q_{28}) \\
&- \tau_{17}q_{17} - \tau_{18}q_{18} - \tau_{27}q_{27} - \tau_{28}q_{28} - \tau_{37}q_{37} - \tau_{38}q_{38} \qquad (2.20)
\end{aligned}
$$

subject to

$$q_{17}+q_{27}+q_{37}+q_{77} \le \bar{S}_7\,, \quad q_{18}+q_{28}+q_{38}+q_{88} \le \bar{S}_8$$

and

$$q_{17},\, q_{27},\, q_{37},\, q_{77},\, q_{18},\, q_{28},\, q_{38},\, q_{88} \ge 0$$

We optimize the new potential function subject to supply constraints. The new equilibrium outcome is that the links from Russia to the Asia Pacific and from the Middle East to Europe carry zero flows, meaning that Russia specializes in the European market and the Middle East specializes in the Asia Pacific. This occurs because Russia has a lower marginal cost of exporting natural gas to Europe, while the Middle East has a lower marginal cost of exporting natural gas to the Asia Pacific.

The equilibrium supply from Russia and the Middle East to Europe decreases to 187.8 Bcm whereas the pre-merger total supply of Russia and the Middle East to Europe was 206.69 Bcm. Similarly, the equilibrium supply to the Asia Pacific from the Middle East is 50.18 Bcm after the merger, whereas the pre-merger total supply of Russia and the Middle East together was 53.39 Bcm. In response, South America, West Africa and North Africa all increase their supply to Europe by decreasing their supply to North America and to their domestic markets. For instance, West Africa decreases supply to North America from 3.1 Bcm to zero and decreases supply to

[32]We label Russia and the Middle East after the merger as 78.

the Asia Pacific from 6.59 Bcm to 6.41 Bcm, but increases supply to Europe from 10.07 Bcm to 14.1 Bcm. Australasia also increases supply to the Asia Pacific by 1 Bcm. The equilibrium price in each region increases due to the decline in equilibrium supply. For instance, the equilibrium price in Europe increases by 4.88 million USD per Bcm due to a 6.9 Bcm decline in total supply. The joint profit of Russia and the Middle East increases by 1.5 billion USD.

If we allow in addition for a flat North American supply curve, then North America becomes a net exporter and exports 23.3 Bcm of natural gas to Europe and 18.1 Bcm of natural gas to the Asia Pacific. Under this scenario, the equilibrium supply of Russia and the Middle East to Europe decreases to 181.2 Bcm and to the Asia Pacific decreases to 44.84 Bcm. With a flat North American supply curve, equilibrium total supply to each region increases, which decreases the equilibrium prices.

The joint profit of Russia and the Middle East increases by 1.5 billion USD compared to no merger scenario. However, with a flat North American supply curve the joint profit of merged pair is 0.08 billion USD less than the reference case[33] (no shale and no collusion) but 0.06 billion USD more than scenario where there is flat North American supply curve and no collusion.

Further analysis on Scenario III

In this section, first we shift the North American supply curve upward and keep its slope constant, and then decrease its slope.

If the intercept of North American supply curve increases to 5, then the equilibrium exports to North America increase by 2.85 Bcm, [34] and the equilibrium price in North America increases by 1.35 million USD per Bcm compared to the case where

[33]This is because of the lower natural gas prices (compared to no exports from North America) in the Asia Pacific and Europe as a result of exports from North America.

[34]The equilibrium export from South America to North America increases by 1.35 Bcm, from West Africa to North America it increases by 0.32 Bcm and from North Africa to North America it increases by 1.18 Bcm.

Russia and the Middle East merge. With this change, equilibrium prices in Europe and the Asia Pacific increase as well.

Next, we assume a flat North American supply curve with an intercept of 20.[35] Under this scenario, North America becomes a net exporter and it exports 0.28 Bcm to the Asia Pacific. In response to a flat North American supply curve with an intercept of 20, all dominant producers shift their supply from North America to Europe and the Asia Pacific. For instance, South America increases supply to Europe from 7.6 Bcm to 13.75 Bcm. In the new equilibrium, prices in Europe decrease by 1.24 million USD per Bcm and decrease in the Asia Pacific by 0.75 million USD per Bcm (compared to the scenario with a merger only).

2.5 Conclusions

This chapter presented a network model of the world natural gas market which consists of consumers, competitive fringe producers, dominant producers and links connecting them. To better mimic the world natural gas market, we represented it under a mixed oligopoly competitive assumption where the North American market is perfectly competitive while the rest of the world consists of oligopolistic and monopolistic markets. We showed that such a noncooperative game has a unique Cournot-Nash equilibrium. We calibrated the model parameters by using production, consumption, price, proved reserves and trade flow data in 2009. This allowed us to quantify the strategic interactions among natural gas producers.

Our scenario analysis focused on the impacts of natural gas exports from North America on domestic and foreign natural gas prices. We find that equilibrium natural gas exports from North America depend on the elasticities of domestic and foreign supplies, the elasticities of domestic and foreign demands, the number of dominant

[35]For any intercept that is higher than 20, North America does not export even when its supply curve is flat. However, it imports when the intercept is 150. More specifically, it imports 3.69 Bcm from South America and 0.69 Bcm from North Africa.

producers that the North American exporter faces in each market and these dominant producers' short run supply capacities. Based on our numerical results, North America exports natural gas when its supply curve is highly elastic and the domestic price impact of its export is very small. Even so, the price impacts on the markets it is exporting to are substantial. We also find that shale gas development in North America decreases dominant producers' market power elsewhere in the world and hence decreases the incentive to form a cartel.

2.6 Appendix

2.6.1 Proof of Theorem 1

Since there is a single price for natural gas in each market, the market with competitive fringe does not export and import. Therefore, there will be some links that become offline when some others are online and vice versa. Therefore, the potential function defined in (2.12) represents a network graph with links that work in the opposite direction.

Let $Z(Q_g^\star) = \{z \in \mathbb{N}_+ : z = \rho(i,j)$ for some (i,j) such that $q_{ij}^\star = 0\}$.[36] Let $|Z(Q_g^\star)| = t^\star$, then $Q^\star_{g-Z(Q_g^\star)}$ is vector of size $r(g) - t^\star$ obtained from $Q_g^\star$ by deleting the zero entries (where $r(g)$ is the size of $Q_g^\star$). It is the vector of equilibrium quantities for links over which there is strictly positive flow from a firm to a market.

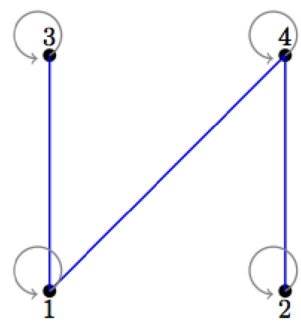
Let $Q_g^\star$ be the equilibrium of the Cournot game at network g. We denote by $g - Z(Q_g^\star$ the network obtained from g by deleting the links which have zero flows at $Q_g^\star$.

Theorem 2 in Ilkilic (2012): Given two networks g and g'. Let $Q_g^\star$ and $Q_{g'}^\star$ be the equilibrium of Cournot game in g and g', respectively. If $g - Z(Q_g^\star) = g' - Z(Q_{g'}^\star)$, then $Q^\star_{g-Z(Q_g^\star)} = Q^\star_{g'-Z(Q_{g'}^\star)}$.

For instance, let the network graph g be:

Graph g:

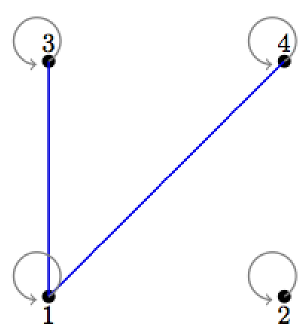
[36]Note that the notation of labeling links is introduced in the proof of Theorem 1 in (1.8.1).



and the flow vector of graph g is $Q_g = \begin{bmatrix} q_{11} & q_{31} & q_{41} & q_{22} & q_{42} & q_{33} & q_{44} \end{bmatrix}$. Suppose that in the equilibrium link from firm 2 to market 4 carries zero flows. Therefore, the equilibrium flow vector is $Q_g^\star = \begin{bmatrix} q_{11}^\star & q_{31}^\star & q_{41}^\star & q_{22}^\star & q_{33}^\star & q_{44}^\star \end{bmatrix}$.

According to Theorem 2 in Ilkilic (2010) solving the equilibrium of network graph g' and g are the same, where g' is

Graph g':



Now, we go back to our optimization problem defined in (2.12).

- If $q_{ie}^\star > 0$ for any $i \in M \setminus \{d_e\}$ means that the exporter f_e is exporting, $\Longrightarrow$ $q_{ej}^\star = 0$ for all $j \in F \setminus \{f_e\}$. As a result (2.12) becomes

$$\begin{aligned}
P^{\star}(\tilde{Q}_g) = & \sum_{d_i \in N_g(f_j)\setminus d_e} \alpha_i \left(\sum_{f_j \in N_g(d_i)} q_{ij} \right) - \sum_{d_i \in N_g(f_j)\setminus d_e} \beta_i \left(\sum_{f_j \in N_g(d_i)} q_{ij}^2 \right) \\
& - \sum_{d_i \in N_g(f_j)\setminus d_e} \beta_i \left(\sum_{1 \le j < k \le n} q_{ij} q_{ik} \right) - \sum_{d_i \in N_g(f_j)\setminus d_j} \sum_{f_j \in N_g(d_i)} \tau_{ij} q_{ij} \\
& - \frac{\beta_e \gamma_e}{\gamma_e + \beta_e} \left(\sum_{d_i \in N_g(f_e)\setminus d_e} q_{ie} \right)^2 - \frac{\alpha_e \gamma_e}{\gamma_e + \beta_e} \sum_{d_i \in N_g(f_e)\setminus d_e} q_{ie} \\
& - \sum_{f_j \in N_g(d_i)\setminus f_e} \frac{\gamma_j}{2} \left(\sum_{d_i \in N_g(f_j)} q_{ij} \right)^2
\end{aligned} \tag{2.21}$$

Following the proof of Theorem 1 in chapter 1, one can show that (2.21) is strictly concave (or $(-1) \times$ (2.21) is strictly convex)[37] and the constraints are linear.

- If $q^{\star}_{ej} > 0$ for any $j \in F \setminus \{f_e\}$ means that the market d_e is importing, $\Longrightarrow$ $q^{\star}_{ie} = 0$ for all $i \in M \setminus \{d_e\}$. Then (2.12) becomes

$$\begin{aligned}
P^{\star}(Q_g) = & \sum_{d_i \in N_g(f_j)\setminus d_e} \alpha_i \left(\sum_{f_j \in N_g(d_i)\setminus f_e} q_{ij} \right) - \sum_{d_i \in N_g(f_j)\setminus d_e} \beta_i \left(\sum_{f_j \in N_g(d_i)\setminus f_e} q_{ij}^2 \right) \\
& - \sum_{d_i \in N_g(f_j)\setminus d_e} \beta_i \left(\sum_{1 \le j < k \le n} q_{ij} q_{ik} \right) - \sum_{d_i \in N_g(f_j)\setminus d_j} \sum_{f_j \in N_g(d_i)\setminus f_e} \tau_{ij} q_{ij} \\
& - \frac{\beta_e \gamma_e}{\gamma_e + \beta_e} \left(\sum_{1 \le j < k \le n} q_{ej} q_{ek} \right) + \frac{\alpha_e \gamma_e}{\gamma_e + \beta_e} \left(\sum_{f_j \in N_g(d_e)\setminus f_e} q_{ej} \right) \\
& - \frac{\beta_e \gamma_e}{\gamma_e + \beta_e} \left(\sum_{f_j \in N_g(d_e)\setminus f_e} q_{ej}^2 \right) - \sum_{f_j \in N_g(d_i)\setminus f_e} \frac{\gamma_j}{2} \left(\sum_{d_i \in N_g(f_j)} q_{ij} \right)^2
\end{aligned} \tag{2.22}$$

[37] *Corollary 2.8* in Zhu (2008) shows that every strictly convex potential game with convex constraints admits a unique equilibrium.

Following the proof of Theorem 1 in chapter 1, one can show that (2.22) is strictly concave and the constraints are linear.

2.6.2 Calibration

We calibrate the model parameters by using production, consumption, price, proved reserves and trade flow data in 2009. We obtain data from BP's Statistical Review of World Energy 2010, EIA and various country websites. In this section, we provide the list of equations that we use in calibration.

The first order conditions of the objective functions of dominant producers' and fringe producers:

Europe:

$$q_{11}: \ \ \alpha_1 - 2\beta_1 q_{11} - \beta_1(q_{12} + q_{14} + q_{15} + q_{16} + q_{17} + q_{18} + q_{19}) - \gamma_1 q_{11} - \mu_{11} - \lambda_1 = 0$$

North American fringe producers:

$$q_{22}: \ \ p_2 - \gamma_2 q_{22} = 0$$

North American exporter:

$$q_{12}: \ \alpha_1 - 2\beta_1 q_{12} - \beta_1(q_{11} + q_{14} + q_{15} + q_{16} + q_{17} + q_{18} + q_{19}) - \tau_{12} - 2\tilde{\beta}_2(q_{12} + q_{32}) - \tilde{\alpha_2} + \tilde{\beta}_2(q_{24} + q_{25} + q_{26} + q_{27} + q_{28} + q_{29}) = 0$$

$$q_{32}: \ \alpha_3 - 2\beta_3 q_{32} - \beta_3(q_{33} + q_{34} + q_{35} + q_{36} + q_{37} + q_{38} + q_{39}) - \tau_{32} - 2\tilde{\beta}_2(q_{12} + q_{32}) - \tilde{\alpha_2} + \tilde{\beta}_2(q_{24} + q_{25} + q_{26} + q_{27} + q_{28} + q_{29}) = 0$$

Asia Pacific:

$$q_{33}: \ \ \alpha_3 - 2\beta_3 q_{33} - \beta_3(q_{32} + q_{34} + q_{35} + q_{36} + q_{37} + q_{38} + q_{39}) - \gamma_3 q_{33} - \mu_{33} - \lambda_3 = 0$$

South America:

$$q_{14}: \quad \alpha_1 - 2\beta_1 q_{14} - \beta_1(q_{11}+q_{12}+q_{15}+q_{16}+q_{17}+q_{18}+q_{19}) - \gamma_4(q_{14}+q_{24}+q_{34}+q_{44}) - \tau_{14} - \mu_{14} - \lambda_4 = 0$$

$$q_{24}: \quad \tilde{\alpha}_2 - 2\tilde{\beta}_2 q_{24} - \tilde{\beta}_2(q_{25}+q_{26}+q_{26}+q_{27}+q_{28}+q_{29}-q_{12}-q_{32}) - \gamma_4(q_{14}+q_{24}+q_{34}+q_{44}) - \tau_{24} - \mu_{24} - \lambda_4 = 0$$

$$q_{34}: \quad \alpha_3 - 2\beta_3 q_{34} - \beta_3(q_{32}+q_{33}+q_{35}+q_{36}+q_{37}+q_{38}+q_{39}) - \gamma_4(q_{14}+q_{24}+q_{34}+q_{44}) - \tau_{34} - \mu_{34} - \lambda_4 = 0$$

$$q_{44}: \quad \alpha_4 - 2\beta_4 q_{44} - \gamma_4(q_{14} + q_{24} + q_{34} + q_{44}) - \mu_{44} - \lambda_4 = 0$$

West Africa:

$$q_{15}: \quad \alpha_1 - 2\beta_1 q_{15} - \beta_1(q_{11}+q_{12}+q_{14}+q_{16}+q_{17}+q_{18}) - \gamma_5(q_{15}+q_{25}+q_{35}+q_{55}) - \tau_{15} - \mu_{15} - \lambda_5 = 0$$

$$q_{25}: \quad \tilde{\alpha}_2 - 2\tilde{\beta}_2 q_{25} - \tilde{\beta}_2(q_{24}+q_{26}+q_{27}+q_{28}+q_{29}-q_{12}-q_{32}) - \gamma_5(q_{15}+q_{25}+q_{35}+q_{55}) - \tau_{25} - \mu_{25} - \lambda_5 = 0$$

$$q_{35}: \quad \alpha_3 - 2\beta_3 q_{35} - \beta_3(q_{32}+q_{33}+q_{36}+q_{37}+q_{38}+q_{39}) - \gamma_5(q_{15}+q_{25}+q_{35}+q_{55}) - \tau_{35} - \mu_{35} - \lambda_5 = 0$$

$$q_{55}: \quad \alpha_5 - 2\beta_5 q_{55} - \gamma_5(q_{15} + q_{25} + q_{35} + q_{55}) - \mu_{55} - \lambda_5 = 0$$

North Africa:

$$q_{16}: \quad \alpha_1 - 2\beta_1 q_{16} - \beta_1(q_{11}+q_{12}+q_{14}+q_{15}+q_{17}+q_{18}+q_{19}) - \gamma_6(q_{16}+q_{26}+q_{36}+q_{66}) - \tau_{16} - \mu_{16} - \lambda_6 = 0$$

$$q_{26}: \quad \tilde{\alpha}_2 - 2\tilde{\beta}_2 q_{26} - \tilde{\beta}_2(q_{24}+q_{25}+q_{27}+q_{28}+q_{29}-q_{12}-q_{32}) - \gamma_6(q_{16}+q_{26}+q_{36}+q_{66}) - \tau_{26} - \mu_{26} - \lambda_6 = 0$$

$$q_{36}: \quad \alpha_3 - 2\beta_3 q_{36} - \beta_3(q_{32}+q_{33}+q_{34}+q_{35}+q_{37}+q_{38}+q_{39}) - \gamma_6(q_{16}+q_{26}+q_{36}+q_{66}) - \tau_{36} - \mu_{36} - \lambda_6 = 0$$

$$q_{66}: \quad \alpha_6 - 2\beta_6 q_{66} - \gamma_6(q_{16} + q_{26} + q_{36} + q_{66}) - \mu_{66} - \lambda_6 = 0$$

Russia:

$$q_{17}: \quad \alpha_1 - 2\beta_1 q_{17} - \beta_1(q_{11}+q_{12}+q_{14}+q_{15}+q_{16}+q_{18}+q_{19}) - \gamma_7(q_{17}+q_{27}+q_{37}+q_{77}) - \tau_{17} - -\mu_{17} - \lambda_7 = 0$$

$$q_{27}: \quad \tilde{\alpha}_2 - 2\tilde{\beta}_2 q_{27} - \tilde{\beta}_2(q_{24}+q_{25}+q_{26}+q_{28}+q_{29}-q_{12}-q_{32}) - \gamma_7(q_{17}+q_{27}+q_{37}+q_{77}) - \tau_{27} - \mu_{27} - \lambda_7 = 0$$

$$q_{37}:\ \ \alpha_3-2\beta_3q_{37}-\beta_3(q_{32}+q_{33}+q_{34}+q_{35}+q_{36}+q_{38}+q_{39})-\gamma_7(q_{17}+q_{27}+q_{37}+q_{77})-\tau_{37}-\mu_{37}-\lambda_7=0$$

$$q_{77}:\ \ \alpha_7-2\beta_7q_{77}-\gamma_7(q_{17}+q_{27}+q_{37}+q_{77})-\mu_{77}-\lambda_7=0$$

Middle East:

$$q_{18}:\ \ \alpha_1-2\beta_1q_{18}-\beta_1(q_{11}+q_{12}+q_{14}+q_{15}+q_{16}+q_{17}+q_{19})-\gamma_8(q_{18}+q_{28}+q_{38}+q_{88})-\tau_{18}-\mu_{18}-\lambda_8=0$$

$$q_{28}:\ \ \tilde{\alpha}_2-2\tilde{\beta}_2q_{28}-\tilde{\beta}_2(q_{24}+q_{25}+q_{26}+q_{27}+q_{29}-q_{12}-q_{32})-\gamma_8(q_{18}+q_{28}+q_{38}+q_{88})-\tau_{28}-\mu_{28}-\lambda_8=0$$

$$q_{38}:\ \ \alpha_3-2\beta_3q_{38}-\beta_3(q_{32}+q_{33}+q_{34}+q_{35}+q_{36}+q_{37}+q_{39})-\gamma_8(q_{18}+q_{28}+q_{38}+q_{88})-\tau_{38}-\mu_{38}-\lambda_8=0$$

$$q_{88}:\ \ \alpha_8-2\beta_8q_{88}-\gamma_8(q_{18}+q_{28}+q_{38}+q_{88})-\mu_{88}-\lambda_8=0$$

Australasia:

$$q_{19}:\ \ \alpha_1-2\beta_1q_{19}-\beta_1(q_{11}+q_{12}+q_{14}+q_{15}+q_{16}+q_{17}+q_{18})-\gamma_9(q_{19}+q_{29}+q_{39}+q_{99})-\tau_{19}-\mu_{19}-\lambda_9=0$$

$$q_{29}:\ \ \tilde{\alpha}_2-2\tilde{\beta}_2q_{29}-\tilde{\beta}_2(q_{24}+q_{25}+q_{26}+q_{27}+q_{28}-q_{12}-q_{32})-\gamma_9(q_{19}+q_{29}+q_{39}+q_{99})-\tau_{29}-\mu_{29}-\lambda_9=0$$

$$q_{39}:\ \ \alpha_3-2\beta_3q_{39}-\beta_3(q_{32}+q_{33}+q_{34}+q_{35}+q_{36}+q_{37}+q_{38})-\gamma_9(q_{19}+q_{29}+q_{39}+q_{99})-\tau_{39}-\mu_{39}-\lambda_9=0$$

$$q_{99}:\ \ \alpha_9-2\beta_9q_{99}-\gamma_9(q_{19}+q_{29}+q_{39}+q_{99})-\mu_{99}-\lambda_9=0$$

Price in Europe:

$$p_1=\alpha_1-\beta_1(q_{11}+q_{12}+q_{14}+q_{15}+q_{16}+q_{17}+q_{18}+q_{19})$$

Price in North America:

$$p_2=\tilde{\alpha}_2-\tilde{\beta}_2(q_{24}+q_{25}+q_{26}+q_{27}+q_{28}+q_{29}-q_{12}-q_{32})$$

Price in Asia Pacific:

$$p_3 = \alpha_3 - \beta_3(q_{32} + q_{33} + q_{34} + q_{35} + q_{36} + q_{37} + q_{38} + q_{39})$$

Price in South America:

$$p_4 = \alpha_4 - \beta_4 q_{44}$$

Price in West Africa:

$$p_5 = \alpha_5 - \beta_5 q_{55}$$

Price in North Africa:

$$p_6 = \alpha_6 - \beta_6 q_{66}$$

Price in Russia:

$$p_7 = \alpha_7 - \beta_7 q_{77}$$

Price in Middle East:

$$p_8 = \alpha_8 - \beta_8 q_{88}$$

Price in Australasia:

$$p_9 = \alpha_9 - \beta_9 q_{99}$$

In 2009, links from North America to Europe, q_{12}, from North America to the Asia Pacific, q_{32}, from South America to the Asia Pacific, q_{34}, from North Africa to the Asia Pacific, q_{36}, from Russia to North America, q_{27}, from the Middle East to North America, q_{28}, from Australasia to Europe, q_{19}, and from Australasia to the Asia Pacific, q_{29}, carried zero flows. Therefore, we do not have any observation to calibrate parameters in the first order conditions of these links. To get the marginal costs of these links, τ_{ij}'s, we used the marginal costs of the links that have (approximately) the same LNG distance.[38] For instance, we use the calibrated value of τ_{15}, from West

[38]Marginal costs of these links are $\tau_{12} = 280$, $\tau_{32} = 300$, $\tau_{34} = 340$, $\tau_{36} = 320$, $\tau_{27} = 340$, $\tau_{28} = 320$, $\tau_{19} = 320$, $\tau_{29} = 340$.

Africa to Europe to approximate for the marginal cost of exporting natural gas from North America to Europe.

We get the equilibrium fringe supply in North America as $q_{22} = \dfrac{p_2}{\gamma_2}$. According to BP's Statistical Review of World Energy 2010, North America supplied 813 Bcm of natural gas to its domestic market at a price of 150 million USD per bcm. By substituting these values into North America's first order condition we get $\gamma_2 = 0.1845$. Next, we define γ_2 as a proportion of proved reserves. In effect, we assume that higher proved reserves[39] indicate lower costs of production. We assume that this proportion holds for each producer hence we define each producer's γ which is the slope of their supply curves in terms of γ_2, the slope of North America's supply curve.

To identify the model parameters uniquely, we assume that North America and Europe have the same choke price. A justification for the assumption is that, North America and Europe have similar technologies for using natural gas.

[39]We obtain data from BP's Statistical Review of World Energy 2010.

2.6.3 Scenario I.a: Change in North America's elasticity of domestic supply

In this section,[40] we analyze the impacts of shale gas developments in North America in the world natural gas market. Unlike the LNG cost parameters that we used in our scenario analysis in (2.4.1), we use the LNG cost parameters in Medlock (2012) and NERA (2012).[41] According to the Table 1 of Medlock (2012), liquefaction cost of 1 mcf (thousand cubic feet) of natural gas in the U.S. is 2.92 USD and the transport cost to United Kingdom is 1.07 USD per mcf and to Japan is 2.15 USD per mcf. For regasification costs, we use the numbers in Figure 58 of NERA (2012). For this analysis, we set the marginal cost of exporting natural gas from North America to Europe to 170 million USD per Bcm and to the Asia Pacific to 210 million USD per Bcm.

In our first experiment, we decreased the slope of North American supply curve by 15 percent. The price in North America decreases from 150 million USD per Bcm to 140.08 million USD per Bcm, and the consumption in North America increases from 828 Bcm to 843.35 Bcm, and its natural gas imports are zero.

Dominant producers which exported to North America in 2009, namely South America, West Africa and North Africa, shift their supply to Europe, the Asia Pacific and their domestic markets. For instance, South America increases its supply to Europe from 7.6 Bcm to 9.2 Bcm, while North Africa increases its supply to Europe from 67.2 Bcm to 67.75 Bcm. As a result, there will be more competition in the European and Asia Pacific markets for Russia, the Middle East and Australasia. In response, they decrease their their supply to Europe and the Asia Pacific. For example, Russia's supply to Europe decreases by 0.56 Bcm and its supply to the Asia Pacific decreases by 0.04 Bcm.

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[40] Equilibrium trade flows under this scenario are provided in Table (2.5).

[41] In 2009, the links from North America to Europe, q_{12}, from North America to the Asia Pacific, q_{32} carried zero flows. Therefore, we do not have any observation to calibrate parameters in the first order conditions of these links. As an experiment, we also use the LNG cost parameters in other studies.

A 15 percent decrease in the slope of North American supply curve is not sufficient for the North American exporter to export natural gas to Europe and/or the Asia Pacific. If we decrease the slope of North American supply curve by 20 percent, then North America exports 2.08 Bcm of natural gas to Europe. With these exports, there will be more competition for all dominant producers in the European market. To prevent a further decline in equilibrium prices, the dominant producers decrease supply to Europe. Under this scenario, equilibrium total supply to Europe increases from 580.3 Bcm to 581 Bcm, which decreases the equilibrium price from 300 million USD per Bcm to 299.25 million USD per Bcm.

All else constant, North America starts exporting when the slope of its supply curve is 20 percent less than that of 2009.

On the other hand, North America stops importing natural gas if we decrease the slope of North American supply curve by more than just 4 percent. With the 4 percent decline in the slope of its supply curve, it imports 0.5 Bcm of natural gas from South America.

Table 2.1 : Network parameters

	Parameter	Value
Intercept of European inverse demand curve:	α_1	711.55
Intercept of North American inverse demand curve:	α_2	711.55
Intercept of residual inverse demand in North America:	$\tilde{\alpha}_2$	152.27
Intercept of Asia Pacific's inverse demand curve:	α_3	691.6
Intercept of South America's inverse demand curve:	α_4	223.2
Intercept of West Africa's inverse demand curve:	α_5	211.4
Intercept of North Africa's inverse demand curve:	α_6	168.5
Intercept of Russian inverse demand curve:	α_7	105.02
Intercept of Middle East's inverse demand curve:	α_8	189.92
Intercept of Australasian inverse demand curve:	α_9	200.47
Slope of European inverse demand curve:	β_1	0.71
Slope of North American inverse demand curve:	β_2	0.67
Slope of residual inverse demand in North America:	$\tilde{\beta}_2$	0.145
Slope of Asia Pacific's inverse demand curve:	β_3	0.94
Slope of South America's inverse demand curve:	β_4	0.69
Slope of West Africa's inverse demand curve:	β_5	10.06
Slope of North Africa's inverse demand curve:	β_6	0.98
Slope of Russian inverse demand curve:	β_7	0.082
Slope of Middle East's inverse demand curve:	β_8	0.26
Slope of Australasian inverse demand curve:	β_9	0.72
Slope of European cost curve:	γ_1	0.332
Slope of North American cost curve:	γ_2	0.184
Slope of Asia Pacific's cost curve:	γ_3	0.358
Slope of South America's cost curve:	γ_4	0.245
Slope of West Africa's cost curve:	γ_5	0.281
Slope of North Africa's cost curve:	γ_6	0.223
Slope of Russian cost curve:	γ_7	0.037
Slope of Middle East's cost curve:	γ_8	0.024
Slope of Australasian cost curve:	γ_9	0.198
Marginal cost of exporting from South America to Europe:	τ_{14}	257.29
Marginal cost of exporting from South America to North America:	τ_{24}	111.58
Marginal cost of exporting from West Africa to Europe:	τ_{15}	283.84
Marginal cost of exporting from West Africa to North America:	τ_{25}	140.98
Marginal cost of exporting from West Africa to Asia Pacific:	τ_{35}	305.2
Marginal cost of exporting from North Africa to Europe:	τ_{16}	220.84
Marginal cost of exporting from North Africa to North America:	τ_{26}	117.77
Marginal cost of exporting from Russia to Europe:	τ_{17}	146.17
Marginal cost of exporting from Russia to Asia Pacific:	τ_{37}	288.74
Marginal cost of exporting from Middle East to Europe:	τ_{18}	271.77
Marginal cost of exporting from Middle East to Asia Pacific:	τ_{38}	265.44
Marginal cost of exporting from Australasia to Asia Pacific:	τ_{39}	197.26

Table 2.2 : Equilibrium trade flows (in Bcm) in Scenario I

Route	2009	Decrease γ_2 by 40%	Flat North American supply curve	Decrease γ_2 by 4%	Decrease γ_2 by 90%
From Europe to Europe	288.10	287.63	285.20	287.63	287.53
From North America to Europe	0.00	0.00	19.71	0.00	0.70
From North America to Asia Pacific	0.00	0.00	17.58	0.00	0.83
From Asia Pacific To Asia Pacific	246.09	245.98	243.45	245.98	245.86
From South America to Europe	7.60	9.26	6.51	9.16	9.15
From South America to North America	7.60	0.00	0.00	0.47	0.00
From South America to Asia Pacific	0.00	0.00	0.00	0.00	0.00
From South America to South America	134.70	135.91	136.33	135.85	135.92
From West Africa to Europe	10.70	10.91	8.99	10.92	10.84
From West Africa to North America	3.09	0.00	0.00	0.00	0.00
From West Africa to Asia Pacific	6.60	7.15	4.89	7.14	7.03
From West Africa to West Africa	10.08	10.11	10.17	10.11	10.11
From North Africa to Europe	67.20	67.76	64.98	67.77	67.65
From North Africa to North America	5.00	0.00	0.00	0.00	0.00
From North Africa to Asia Pacific	0.00	0.00	0.00	0.00	0.00
From North Africa to North Africa	69.20	69.65	69.93	69.64	69.66
From Russia to Europe	181.10	180.54	177.60	180.55	180.45
From Russia to North America	0.00	0.00	0.00	0.00	0.00
From Russia to Asia Pacific	6.20	6.16	3.14	6.16	6.03
From Russia to Russia	485.49	486.10	488.78	486.08	486.31
From Middle East to Europe	25.59	24.91	21.57	24.92	24.78
From Middle East to North America	0.00	0.00	0.00	0.00	0.00
From Middle East to Asia Pacific	47.19	47.06	43.74	47.06	46.90
From Middle East to Middle East	345.58	345.61	345.91	345.61	354.62
From Australasia to Europe	0.00	0.00	0.00	0.00	0.00
From Australasia to North America	0.00	0.00	0.00	0.00	0.00
From Australasia to Asia Pacific	88.30	88.18	85.24	88.18	88.04
From Australasia to Australasia	110.80	110.82	111.17	110.81	110.83

Table 2.3 : Equilibrium trade flows (in Bcm) in Scenario II

Route	2009	Decrease γ_2 by 40%	Decrease γ_2 by 75 % w/ exports	Decrease γ_2 by 75% w/o exports	Flat North American supply curve
From Europe to Europe	288.10	288.10	288.10	288.10	287.07
From North America to Europe	0.00	0.00	0.00	0.00	22.45
From North America to Asia Pacific	0.00	0.00	3.75	0.00	39.96
From Asia Pacific To Asia Pacific	246.09	246.10	246.10	246.10	246.10
From South America to Europe	7.60	14.26	14.12	14.26	8.62
From South America to North America	7.60	0.00	0.00	0.00	0.00
From South America to Asia Pacific	0.00	0.00	0.00	0.00	0.00
From South America to South America	134.70	135.16	135.18	135.16	136.01
From West Africa to Europe	10.70	0.00	0.00	0.00	1.27
From West Africa to North America	3.09	0.00	0.00	0.00	0.00
From West Africa to Asia Pacific	6.60	21.22	21.17	21.22	19.41
From West Africa to West Africa	10.08	9.26	9.31	9.26	9.80
From North Africa to Europe	67.20	72.40	72.36	72.40	67.12
From North Africa to North America	5.00	0.00	0.00	0.00	0.00
From North Africa to Asia Pacific	0.00	0.00	0.00	0.00	0.00
From North Africa to North Africa	69.20	69.00	69.04	69.00	69.72
From Russia to Europe	181.10	180.61	180.67	180.61	176.74
From Russia to North America	0.00	0.00	0.00	0.00	0.00
From Russia to Asia Pacific	6.20	33.72	32.66	33.72	22.82
From Russia to Russia	485.49	458.47	459.47	458.47	473.24
From Middle East to Europe	25.59	19.17	19.45	19.17	18.55
From Middle East to North America	0.00	0.00	0.00	0.00	0.00
From Middle East to Asia Pacific	47.19	70.25	69.35	70.25	61.79
From Middle East to Middle East	345.58	328.98	329.60	328.98	338.06
From Australasia to Europe	0.00	0.00	0.00	0.00	0.00
From Australasia to North America	0.00	0.00	0.00	0.00	0.00
From Australasia to Asia Pacific	88.30	100.98	100.49	100.98	95.68
From Australasia to Australasia	110.80	98.12	98.61	98.12	103.42

Table 2.4 : Equilibrium trade flows (in Bcm) in Scenario III

Route	2009	Russia and Middle East merger, without shale	Russia and Middle East merger, with shale
From Europe to Europe	288.10	288.10	287.67
From North America to Europe	0.00	0.00	23.33
From North America to Asia Pacific	0.00	0.00	18.19
From Asia Pacific To Asia Pacific	246.09	246.10	243.89
From South America to Europe	7.60	11.92	9.31
From South America to North America	7.60	4.58	0.00
From South America to Asia Pacific	0.00	0.00	0.00
From South America to South America	134.70	133.39	135.91
From West Africa to Europe	10.70	14.10	11.63
From West Africa to North America	3.09	0.00	0.00
From West Africa to Asia Pacific	6.60	6.42	4.75
From West Africa to West Africa	10.08	9.96	10.14
From North Africa to Europe	67.20	71.46	67.81
From North Africa to North America	5.00	1.68	0.00
From North Africa to Asia Pacific	0.00	0.00	0.00
From North Africa to North Africa	69.20	68.26	69.65
From Russia to Europe	181.10	187.85	181.20
From Russia to North America	0.00	0.00	0.00
From Russia to Asia Pacific	6.20	0.00	0.00
From Russia to Russia	485.49	484.95	488.69
From Middle East to Europe	25.59	0.00	0.00
From Middle East to North America	0.00	0.00	0.00
From Middle East to Asia Pacific	47.19	50.18	44.84
From Middle East to Middle East	345.58	346.58	346.81
From Australasia to Europe	0.00	0.00	0.00
From Australasia to North America	0.00	0.00	0.00
From Australasia to Asia Pacific	88.30	89.26	85.75
From Australasia to Australasia	110.80	109.84	111.11

Table 2.5 : Equilibrium trade flows (in Bcm) in Scenario I.a

Route	2009	Decrease γ_2 by 20 %	Decrease γ_2 by 15 %	Decrease γ_2 by 4 %
From Europe to Europe	288.10	287.38	287.63	287.64
From North America to Europe	0.00	2.08	0.00	0.00
From North America to Asia Pacific	0.00	0.00	0.00	0.00
From Asia Pacific To Asia Pacific	246.09	245.97	245.98	245.98
From South America to Europe	7.60	8.98	9.26	9.16
From South America to North America	7.60	0.00	0.00	0.50
From South America to Asia Pacific	0.00	0.00	0.00	0.00
From South America to South America	134.70	135.96	135.91	135.85
From West Africa to Europe	10.70	10.64	10.91	10.93
From West Africa to North America	3.10	0.00	0.00	0.00
From West Africa to Asia Pacific	6.60	7.19	7.15	7.14
From West Africa to West Africa	10.08	10.12	10.11	10.11
From North Africa to Europe	67.20	67.48	67.76	67.78
From North Africa to North America	5.00	0.00	0.00	0.00
From North Africa to Asia Pacific	0.00	0.00	0.00	0.00
From North Africa to North Africa	69.20	69.68	69.65	69.65
From Russia to Europe	181.10	180.24	180.54	180.55
From Russia to North America	0.00	0.00	0.00	0.00
From Russia to Asia Pacific	6.20	6.19	6.16	6.16
From Russia to Russia	485.49	486.37	486.10	486.09
From Middle East to Europe	25.59	24.56	24.91	24.93
From Middle East to North America	0.00	0.00	0.00	0.00
From Middle East to Asia Pacific	47.19	47.05	47.06	47.07
From Middle East to Middle East	345.58	345.63	345.61	345.61
From Australasia to Europe	0.00	0.00	0.00	0.00
From Australasia to North America	0.00	0.00	0.00	0.00
From Australasia to Asia Pacific	88.30	88.17	88.18	88.18
From Australasia to Australasia	110.80	110.82	110.82	110.82

Chapter 3

Strategic capacity investments in an imperfectly competitive world natural gas market

3.1 Introduction

This chapter extends the first chapter, which solved for the constrained Cournot equilibrium with n producers, each having a fixed supply capacity, and m consumers connected through a bipartite network. More specifically, in this chapter we relax the assumption that producers have fixed supply capacities and instead allow them to invest in their supply capacities.

Models of capacity expansion in oligopolistic markets have tended to be applied most to electricity markets. This is because the perfect competition assumption is a strong one when it comes to restructured electricity markets. Even though there are several studies[1] looking at the operations of oligopolistic electricity markets, the literature on strategic investments in these markets is relatively new. Electricity market models dealing with both investments and operations start with Murphy and Smeers (2005), who considered three models of investment in generation capacity. The first model assumed perfect competition. The second model extended the Cournot model to include investments in new generation capacities, where capacity is simultaneously built and sold in long-term contracts (open-loop Nash equilibrium). The third model separated the investment and sales decision, assuming investment decisions are made in the first stage and sales in the second stage (closed-loop Nash equilibrium). Murphy and Smeers (2005) considered a simple electricity system where all demand and supply is concentrated at a single node and there are two generators behaving strategi-

[1] Among others, see Wei and Smeers (1999), Daxhelet and Smeers (2001).

cally. The most of the subsequent studies looked at the strategic investment problem in a duopolistic market. For instance, Ehrenmann and Smeers (2006) developed a two stage capacity expansion game under the assumption of duopoly. Their model was similar to Murphy and Smeers (2005), but unlike them, Ehrenmann and Smeers (2006) assumed no uncertainty. Genc and Zacoor (2010) extended the Murphy and Smeers (2005) two stage model to a dynamic duopoly with capacity investments under demand uncertainty. Genc and Zacoor (2010) characterized all the open-loop and closed-loop Nash equilibria of this game.

Ventosa et al. (2002) extended the capacity expansion problem from a duopolistic electricity market to an oligopolistic electricity market. However, they retained the assumption of a single demand node. They presented two approaches. In the first approach, firms choose their output and generating capacity under the assumption of Cournot competition. In the second approach, a “leader firm” chooses its capacity in the first stage, as in the Stackelberg game, and then in a second stage all the firms compete in quantity and capacity as in the Cournot game.

Our model adds to the strategic capacity investment literature by allowing for Cournot competition in a networked market with multiple demand nodes and multiple suppliers. However, our model makes a simplifying assumption that the network graph is fixed. A future extension of this chapter would look for an equilibrium in a dynamic network graph with demand uncertainties over multiple periods. This is a difficult problem. There is even a conceptual issue in the problem with multiple periods. Hartley and Kyle (1989) show that there can be multiple equilibria depending on what new investors conjecture about future investor behavior.

We modify the first chapter by allowing producers to invest in their supply capacities before making their production decisions. We show that this game can also be represented as a potential game and the open-loop Cournot-Nash[2] equilibrium and

[2]According to Fudenberg and Levine (1988), in the open-loop, players cannot observe the play of their opponents. In the closed-loop equilibrium, all past play is common knowledge at the beginning of each stage. Following their definition, in this chapter we assume that in the open-loop producers

the closed-loop Cournot-Nash equilibrium of this potential game coincide. We apply this model to a world natural gas network formed by using BP's Statistical Review of World Energy, 2010. We then consider various changes to the basic model in a number of scenarios. We focus on how to look for strategic investment decisions.

In Section 3.2, we define our open-loop Cournot-Nash game model and solve for its unique Cournot-Nash equilibrium. Section 3.3 is devoted to analyzing different policy scenarios. The chapter concludes in Section 3.4. In Section 3.5, we introduce a dynamic game which would be a future extension of this chapter. In the appendix, we introduce the closed-loop Cournot-Nash game and also calibrate the model parameters.

3.2 Model

In this section,[3] we introduce the open-loop Cournot-Nash game, where capacity investment and production decisions are made simultaneously. In the appendix, we introduce the closed-loop Cournot-Nash game[4] show that in a two stage game with no uncertainty, its equilibrium coincides with the equilibrium of the open-loop Cournot-Nash game.

Following the first chapter, we assume that markets have linear inverse demand functions. Given a market d_i and a flow vector Q_g the price, p_i, at d_i is

$$p_i(Q_g) = \alpha_i - \beta_i h_i, \tag{3.1}$$

where α_i and β_i are positive constants and h_i is natural gas consumption in market

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do not know their competitiors' decisions in supply capacity investments and their current supply decisions, while in the closed-loop equilibrium they do know about the past plays of their competitiors, i.e, supply capacity investments, but they do not know about their competitors' current supply decisions.

[3]We use the same notation as in the first chapter.

[4]An assumption that the supply capacity investment is not productive instantly, meaning that there is a lag between a producer's capacity investment and the production, would be equivalent to solving the closed-loop Cournot-Nash equilibrium. For further details, see (3.6.1).

d_i:

$$h_i = \sum_{f_j \in N_g(d_i)} q_{ij} \tag{3.2}$$

We assume that the natural gas producer has zero costs of production in the short run up to its production capacity, $\bar{S}_j = \bar{S}_j^0 + k_j$,[5] and the marginal cost of capacity investment is constant and positive, θ_j.[6] Therefore, the cost of expanding production capacity by k_j is equal to $\theta_j k_j$.

We also assume that cost of exporting natural gas is proportional to the export volume. Therefore, for firm f_j the short-run total cost of exporting is

$$T_j(Q_g) = \sum_{d_i \in N_g(f_j) \setminus d_j} \tau_{ij} q_{ij}, \tag{3.3}$$

where τ_{ij} is the marginal cost of exporting natural gas to market i.

Firm j's total supply is denoted as s_j:

$$s_j = \sum_{d_i \in N_g(f_j)} q_{ij}, \tag{3.4}$$

where $s_j \leq \bar{S}_j = \bar{S}_j^0 + k_j$.

Given a graph Q_g and a supply capacity of $\bar{S}_j$, firm j maximizes its profit by choosing q_{ij} and k_j. Then, the potential function of this game is:

$$\begin{aligned} P^\star(Q_g) = & \sum_{d_i \in N_g(f_j)} \alpha_i \left(\sum_{f_j \in N_g(d_i)} q_{ij} \right) - \sum_{d_i \in N_g(f_j)} \beta_i \left(\sum_{f_j \in N_g(d_i)} q_{ij}^2 \right) \\ & - \sum_{d_i \in N_g(f_j)} \beta_i \left(\sum_{1 \leq j < k \leq n} q_{ij} q_{ik} \right) - \sum_{d_i \in N_g(f_j) \setminus d_j} \sum_{f_j \in N_g(d_i)} \tau_{ij} q_{ij} - \sum_{f_j} \theta_j k_j \end{aligned} \tag{3.5}$$

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[5] $\bar{S}_j^0$ is the starting capacity at the beginning of period 0 and k_j is the capacity expansion in that period.

[6] In the calibration, we approximate θ_j by using the inverse of the reserve to production ratio, $\left(\frac{R}{P}\right)^{-1}$.

subject to

$$\bar{S}_j^0 + k_j \geq \sum_{d_i \in N_g(f_j)} q_{ij} \quad \text{for all} \;\; j \in F \tag{3.6}$$

and

$$q_{ij} \geq 0 \quad \text{for all} \;\; (i,j) \in g \tag{3.7}$$

and

$$k_j \geq 0 \quad \text{for all} \;\; j \in F. \tag{3.8}$$

It can be verified that for every link from firm j to market i, that is q_{ij}, and for every link that is not from firm j to market i, that is q_{-ij},

$$\pi_j(q_{ij}, q_{-ij}) - \pi_j(x_{ij}, q_{-ij}) = P^\star(q_{ij}, q_{-ij}) - P^\star(x_{ij}, q_{-ij}) \tag{3.9}$$

and for every firm j's capacity investment, that is k_j, and for every firm's, that is not firm j, capacity investment, that is k_{-j}, $P^\star(Q_g)$[7] satisfies

$$\pi_j(k_j, k_{-j}) - \pi_j(t_j, k_{-j}) = P^\star(k_j, k_j) - P^\star(t_j, k_{-j}) \tag{3.10}$$

A function $P^\star$ satisfying (3.9) and (3.10) is called a potential function, which requires

$$\frac{\partial \pi_j}{\partial q_{ij}} = \frac{\partial P^\star}{\partial q_{ij}} \quad \text{for all} \;\; (i,j) \in g \tag{3.11}$$

and

$$\frac{\partial \pi_j}{\partial k_j} = \frac{\partial P^\star}{\partial k_j} \quad \text{for all} \;\; (j) \in g \tag{3.12}$$

Under no uncertainty, choosing capacity investment and production amounts to the same thing as choosing capacity investment in the first stage and choosing pro-

[7] Q_g is the vector of quantities in graph g.

duction in the second stage.[8]

$$\mathcal{L} = \sum_{d_i \in N_g(f_j)} \alpha_i \left(\sum_{f_j \in N_g(d_i)} q_{ij} \right) - \sum_{d_i \in N_g(f_j)} \beta_i \left(\sum_{f_j \in N_g(d_i)} q_{ij}^2 \right) - \sum_{d_i \in N_g(f_j)} \beta_i \left(\sum_{1 \leq j < k \leq n} q_{ij} q_{ik} \right)$$
$$- \sum_{d_i \in N_g(f_j) \backslash d_j} \sum_{f_j \in N_g(d_i)} \tau_{ij} q_{ij} + \sum_{f_j \in N_g} \lambda_j \left(\bar{S}_j + k_j - \sum_{d_i \in N_g(f_j)} q_{ij} \right) + \sum_{f_j \in N_g} \mu_j k_j$$
$$+ \sum_{d_i \in N_g(f_j)} \sum_{f_j \in N_g(d_i)} \iota_{ij} q_{ij} \tag{3.13}$$

There exists $\lambda_j^\star$, $\mu_j^\star$ and $\iota_{ij}^\star$ such that $q_{ij}^\star$, $\lambda_j^\star$, $\mu_j^\star$ and $\iota_{ij}^\star$ that satisfy the following Kuhn-Tucker optimality conditions:

$$\frac{\partial \mathcal{L}}{\partial q_{ij}} = \alpha_i - 2\beta_i q_{ij} - \beta_i \left(\sum_{f_k \in N_g(d_i) \backslash \{f_j\}} q_{ik} \right) - \lambda_j - \tau_{ij} - \iota_{ij} = 0 \tag{3.14a}$$

$$\frac{\partial \mathcal{L}}{\partial k_j} = -\theta_j + \lambda_j + \mu_j = 0 \tag{3.14b}$$

$$\frac{\partial \mathcal{L}}{\partial \lambda_j} = \bar{S}_j^0 + k_j - \sum_{d_i \in N_g(f_j)} q_{ij} \geq 0 \tag{3.14c}$$

$$\frac{\partial \mathcal{L}}{\partial \mu_j} = k_j \geq 0 \tag{3.14d}$$

$$\frac{\partial \mathcal{L}}{\partial \iota_{ij}} = q_{ij} \geq 0 \tag{3.14e}$$

$$\lambda_j \frac{\partial \mathcal{L}}{\partial \lambda_j} = \lambda_j \left(\bar{S}_j^0 + k_{ij} - \sum_{d_i \in N_g(f_j)} q_{ij} \right) = 0 \tag{3.14f}$$

$$\mu_j \frac{\partial \mathcal{L}}{\partial \mu_j} = \mu_j k_j = 0 \tag{3.14g}$$

[8]See Section (3.6.1) for an analysis of the two stage capacity investment and optimal production game.

$$\iota_{ij}\frac{\partial \mathcal{L}}{\partial \iota_{ij}} = \iota_{ij}q_{ij} = 0 \tag{3.14h}$$

Therefore, equilibrium trade flow from firm j to market i is[9]

$$q^{\star}_{ij} = \begin{cases} \dfrac{\alpha_i - \tau_{ij} - \lambda_j - \beta_i\Bigg(\displaystyle\sum_{f_k \in N_g(d_i)\backslash\{f_j\}} q_{ik}\Bigg)}{2\beta_i} & \text{if } \frac{\partial \pi_j}{\partial q_{ij}}\Big|_{Q_g} \geq 0 \\ 0 & \text{if } \frac{\partial \pi_j}{\partial q_{ij}}\Big|_{Q_g} < 0 \end{cases} \tag{3.15}$$

and if $k_j > 0$ then $\mu_j = 0 \implies \theta_j = \lambda_j \implies \dfrac{\partial C(k_j)}{\partial k_j} = \lambda^{\star}_j$.

Theorem 1: The Cournot game has a unique Nash equilibrium.

Proof: Proof of Theorem 1 in chapter 1 applies to the proof this theorem. Note that the constraints of this game are linear functions of the new choice variables that we introduced in this chapter, k_j.

Proposition 1: When there is no uncertainty, the open-loop Cournot-Nash equilibrium and the closed-loop Nash equilibrium investments coincide.

Proof: See Section (3.6.1) in the appendix.

3.3 Scenario analysis

In this section, we analyze the same scenarios[10] as in the first chapter by using the same methodology.[11]

3.3.1 Scenario I: Increased competition between Russia and the Middle East

In this scenario, we consider bringing Iraqi gas to the European market via a pipeline through Turkey, "Nabucco" pipeline. We incorporate this scenario into our model by

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[9] Note that, for $i = j$, $\tau_{ij} = 0$.

[10] We do not consider the scenarios with an exogenous change in supply capacity.

[11] Equilibrium trade flows and supply capacity investments are provided in Table (3.2) and Table (3.3).

using the RWGTM's cost estimate[12] for pipeline from Iraq to Istanbul, Istanbul to Bulgaria and Bulgaria to Austria. We get the marginal cost of exporting to Europe by taking the weighted average[13] of marginal costs of exporting natural gas to Europe via pipeline and via LNG[14], which decreases to 237.97 million USD. With this reduction, the Middle East increases supply to Europe from 25.6 Bcm to 56.36 Bcm by expanding its supply capacity by 30.68 Bcm which is 7.3 percent of its supply capacity in 2009. Unlike the fixed capacity scenario, the Middle East does not simply reallocate its resources.[15] When Nabucco is built there will be more competition in the European market for all producers that are connected to it: Europe, South America, West Africa, North Africa and Russia. They will decrease their supply to Europe to avoid further decline in the equilibrium natural gas price in Europe. For instance, South America's equilibrium supply to Europe decreases from 7.6 Bcm to 2.47 Bcm. Similarly, Russia's equilibrium supply decreases from 181.1 Bcm to 175.97 Bcm.

Under this scenario, equilibrium total supply to Europe increases from 580.3 Bcm to 585.43 Bcm, which decreases the equilibrium price in Europe from 300 million USD per Bcm to 294 million USD per Bcm. Since, producers do not shift their resources between markets, neither the consumption nor the price changes in North America and the Asia Pacific.

Under this scenario, profits of all producers that are connected to Europe (except the profits of the Middle East) decline. This is due to a 6 million USD per Bcm decline in the European price. Profits of the Middle East are 0.75 billion USD higher than its profits with fixed supply capacity. With capacity investments, the Middle East is able to increase supply to Europe without shifting supply from the Asia Pacific and domestic markets.

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[12]We consider tariffs paid to transit countries plus the operating and maintenance costs.

[13]This scenario assumes that 20 percent of natural gas is carried via pipeline and 80 percent is carried via LNG.

[14]The cost of exporting natural gas via LNG is calibrated in the previous section.

[15]This result changes as the marginal cost of expanding supply capacity changes.

3.3.2 Scenario II: Decreased competition between Russia and the Middle East

In this scenario, Russia and the Middle East[16] collude to maximize their joint profits. Given the natural gas network we had in 2009, the joint profit of Russia and the Middle East[17] after collusion is

$$\begin{aligned}\Pi_{78}(Q_g) =& \alpha_1(q_{17}+q_{18}) - \beta_1(q_{11}+q_{14}+q_{15}+q_{16}+q_{17}+q_{18})(q_{17}+q_{18}) + \alpha_3(q_{37}+q_{38})\\ &- \beta_3(q_{33}+q_{35}+q_{37}+q_{38}+q_{39})(q_{37}+q_{38}) - \tau_{17}q_{17} - \tau_{18}q_{18} - \tau_{37}q_{37} - \tau_{38}q_{38}\\ &- \theta_7 k_7 - \theta_8 k_8 \qquad (3.16)\end{aligned}$$

subject to

$$q_{17}+q_{37}+q_{77} \le \bar{S}_7 + k_7\ , \quad q_{18}+q_{38}+q_{88} \le \bar{S}_8 + k_8$$

and

$$q_{17},\, q_{37},\, q_{77},\, q_{18},\, q_{38},\, q_{88},\, k_7,\, k_8 \ge 0. \qquad (3.17)$$

After the merger, Russia and the Middle East reduce their combined output and their equilibrium supplies to each of the markets they share, namely Europe and the Asia Pacific. The new equilibrium outcome is that the links from Russia to the Asia Pacific and from the Middle East to Europe carry zero flows. This occurs because Russia has a lower marginal cost of exporting natural gas to Europe, while the Middle East has a lower marginal cost of exporting natural gas to the Asia Pacific.

The equilibrium supply of Russia and the Middle East to Europe is 185.38 Bcm after the merger. The pre-merger supply from Russia to Europe was 181.1 Bcm and from the Middle East to Europe was 25.6 Bcm. Similarly, the equilibrium supply of Russia and the Middle East to the Asia Pacific is 48.45 Bcm after the merger. The pre-merger supply from Russia to the Asia Pacific was 6.2 Bcm and from the Middle East to the Asia Pacific was 47.19 Bcm.

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[16] We call this a "merger" between Russia and the Middle East.

[17] We label the merged Russia and Middle East supplier as 78.

As a result of collusion, prices rise in both Europe and the Asia Pacific. In the new equilibrium, total supply to Europe decreases from 580.3 Bcm to 576 Bcm, which increases the equilibrium price from 300 million USD per Bcm to 304.45 million USD per Bcm. In the new equilibrium, total supply to the Asia Pacific decreases from 394.39 Bcm to 393.1 Bcm, which increases the equilibrium price in the Asia Pacific from 320 million USD per Bcm to 321.62 million USD per Bcm.

Neither the consumption nor the equilibrium price change in North America. This is because producers increase their supply to Europe and the Asia Pacific by expanding their supply capacity and not by shifting supplies from other markets.

In response to Russian and Middle Eastern collusion, other suppliers connected to Europe and the Asia Pacific invest in their supply capacities to increase their supply to Europe and the Asia Pacific. For instance, West Africa increases its supply capacity by 5.52 Bcm, which is approximately 18.13 percent of its supply capacity in the reference case. Its capacity investment is the highest of all other firms supplying Europe or the Asia Pacific. That is because it is the only producer that is connected to *both* Europe and the Asia Pacific. West Africa increases supply to Europe by 4.27 Bcm and to the Asia Pacific by 1.24 Bcm and expands its supply capacity by 5.52 Bcm.

The joint profit of Russia and the Middle East increases by 1.06 billion USD compared to total joint profits in 2009. However, their joint profit decreases by 1.12 billion USD compared to a scenario with a merger but holding supply capacities fixed. The impact of such collusion would be more dramatic on the equilibrium prices if producers were constrained by their supply capacities.

3.3.3 Scenario III: An increase in Asia Pacific's natural gas demand

According to the IEA's 2010 World Energy Outlook, China's demand is projected to grow faster than any other region, at an average of almost 6 percent per year 2008-2035. The IEA report projects that from 2008 to 2015 Asia's demand will grow from

341 Bcm to 497 Bcm a year.

The expected demand increase in the Asia Pacific is incorporated into our model by increasing the choke price in the Asia Pacific by 5 percent. In response, all producers that are connected the Asia Pacific increase supplies by expanding their supply capacities. Hence, the total production in the Asia Pacific, West Africa, Russia, the Middle East and Australasia increase.

West Africa expands its supply capacity by 5.32 Bcm which corresponds to 17.4 percent of its supply capacity in 2009. On the other hand, Russia increases its supply capacity by 5.22 Bcm which is around 0.77 percent of its supply capacity in 2009.

With the increase in Asia Pacific's demand, equilibrium supply to the Asia Pacific increases from 394.34 Bcm to 421.07 Bcm and the equilibrium price increases from 320 million USD per Bcm to 326 million USD per Bcm. Neither the consumption nor the equilibrium price change in any other region.

The increase in the Asia Pacific price increases the profits of producers connected to the Asia Pacific. Moreover, the Asia Pacific makes more profit under this scenario than the scenario with fixed supply capacities. Under fixed supply capacities, the Asia Pacific is not able expand its supply capacity to respond to an increase in the demand for natural gas in its domestic market. On the other hand, all other dominant producers make more profits with the fixed supply capacities, as the equilibrium prices in all three importing markets were higher.

3.3.4 Scenario IV: Increase in importers' natural gas demand

In this scenario, we consider an increase in demand for natural gas from all importing countries. According to IEA's Energy Outlook, global demand for natural is projected to increase by 50 percent to 5 trillion cubic meters in 2035.[18]

These demand increases are incorporated into our model by increasing the choke prices in Europe, North America and Asia Pacific by 2 percent. With a 2 percent

[18] See http://www.iea.org/newsroomandevents/pressreleases/2012/november/name,33015,en.html

increase in the choke prices, all producers invest in their supply capacities in order to increase supplies to importing regions. For instance, West Africa expands its supply capacity by 11.29 Bcm, which corresponds to 37 percent of its supply capacity in 2009. On the other hand, Russia expands its supply capacity by 4.51 Bcm, which corresponds to 0.67 percent of its supply capacity in 2009.

Under this scenario, total supplies to Europe, North America and the Asia Pacific increase. Due to the increase in demand, the equilibrium prices in the importing regions increase. For instance, total supply to Europe increases by 15.17 Bcm and the equilibrium price increases by 2.6 million USD per Bcm. Similarly, total supply to North America increases by 26.13 Bcm and the equilibrium price increases by 1.6 million USD per Bcm. The equilibrium price in the Asia Pacific increases by 2.8 million USD per Bcm due to the increase in total supply by 10.66 Bcm.

The profit of each producer increases as the equilibrium prices in the importing regions increase. All producers would make more profits if there were fixed supply capacities, or if they cooperated and did not expand their supply capacities. However, it is hard to maintain such a cooperative behavior as cheating is profitable.

3.3.5 Scenario V: Russia to China pipeline

In this scenario, we assume that Western Siberia and China are connected through a pipeline. To incorporate this into our model, we use the RWGTM's cost estimates for pipeline routes from West Siberia to China. We assume that 30 percent of natural gas from Russia to the Asia Pacific is carried via pipeline and 70 percent is carried via LNG.

If 30 percent of natural gas is carried via pipeline, the marginal cost of exporting one Bcm of natural gas from Russia to the Asia Pacific decreases by 74.72 million USD per Bcm. With this reduction, Russia increases supply to the Asia Pacific from 6.19 Bcm to 53.5 Bcm. Russia meets this supply to the Asia Pacific by increasing its supply capacity by 47.3 Bcm. When a Russia-China pipeline is built there will be more competition in the Asia Pacific for all producers that are connected to it: the Asia Pacific, West Africa, the Middle East and Australasia. In response, they decrease their supply to the Asia Pacific. For instance, equilibrium supply from Australasia to the Asia Pacific decreases by 10.17 Bcm.

Under this scenario, neither the consumption nor the equilibrium prices change in Europe and North America. This is because Russia increases supply to the Asia Pacific without displacing supplies from other markets. However, total supply to the Asia Pacific increases by 10.17 Bcm, which decreases the equilibrium price by 13.2 million USD per Bcm. If there were no capacity expansions, Russia's equilibrium supply to the Asia Pacific would be 5.65 Bcm lower and hence the equilibrium prices would be 1.83 million USD per Bcm higher.

Under this scenario, profit of Russia increases by 3.66 billion USD, which is 1.53 billion USD more than its profits increase when supply capacities are fixed. All other producers connected to the Asia Pacific make less profits both because their market share decreases and also because the equilibrium price in Asia Pacific declines. On the other hand, producers that are not connected to the Asia Pacific make the same profits as in 2009. This is because equilibrium trade flows in these links do not change. Therefore, equilibrium prices remain unchanged. However, other producers prefer a scenario where the Russia-China pipeline is built and the supply capacities are fixed to this scenario. Under the fixed supply capacities, Russia displaces its supplies from Europe and its domestic market to the Asia Pacific which increases prices in the European market.

3.4 Conclusions

In this chapter, we solved for the equilibrium strategic capacity investments and trade flows in a network model of the world natural gas market which consists of consumers, producers (which are represented as strategic Cournot players) and links connecting them. We assumed a two period model with no uncertainty and showed that this game has a unique Nash equilibrium. We also showed that the open-loop Cournot-Nash equilibrium and closed-loop Cournot-Nash equilibrium investments of this game coincide. Our chapter contributes to the literature in strategic capacity investments by allowing for Cournot competition in a networked market with multiple demand nodes and multiple suppliers. In this chapter, we assume that the strategic capacity investments are continuous. However, in reality economies of scale will make the capacity investments lumpy. A good extension of this chapter would follow Hartley and Kyle (1989), where demand grows smoothly over time and the investment is the only cost which has a fixed size. In their paper, they show that there is an efficient investment path which is a function of the investment sequence and investment times. They also show the oligopolistic market can have multiple equilibria depending on what investors believe about future investment decisions. Similar problem can be applied to this network problem to solve for the strategic investment path with lumpiness.

We looked at the same scenarios as we looked at in our first chapter and compared the results. We find that producers respond to changes in market conditions by investing in their supply capacities instead of displacing their resources from other markets.

3.5 Future research

A future extension of this problem would involve a dynamic network. A way to do this is to allow for a complete network graph; an m market n producer Cournot game requires $m \times n$ links connecting them. In each stage, some links may come online, while others may not be used. We assume that each link has a fixed flow capacity. Each producer could a fixed supply capacity in the short-run, but can invest in link and supply capacities. However, these investments are not productive instantly. Therefore, in each period, producer j chooses how much to supply to each market that it is connected to and how much to invest in its links' flow capacities and its supply capacity for future periods. We also assume that producer j's current capacities depreciate from this period to next period at a fixed rate of η_j for its supply capacity and at a fixed rate of ι_{ij} for its links' flow capacity.

The Bellman equation of such a potential game would be:

$$
\begin{aligned}
V(\bar{S}_j^t, \bar{q}_{ij}^t) = \max_{q_{ij}^t, k_j^t, x_{ij}^t} & \sum_{d_i} \alpha_i \sum_{f_j} q_{ij}^t - \sum_{d_i} \beta_i \sum_{f_j} (q_{ij}^t)^2 - \sum_{d_i} \beta_i \left(\sum_{1 \le j < k \le n} q_{ij}^t q_{ik}^t \right) \\
& - \sum_{d_i} \sum_{f_j} \tau_{ij} q_{ij}^t - \sum_{f_j} \theta_j k_j^t - \sum_{d_i} \sum_{f_j} \kappa_{ij} x_{ij}^t + \rho V(\bar{S}_j^{t+1}, \bar{q}_{ij}^{t+1})
\end{aligned}
\tag{3.18}
$$

subject to

$$\sum_{d_i} q_{ij}^t \le \bar{S}_j^t \qquad \text{and the Lagrange multiplier is } \lambda_j^t \tag{3.19a}$$

$$\bar{q}_{ij}^t \ge q_{ij}^t \quad \text{for all } \ (i,j) \qquad \text{and the Lagrange multiplier is } \mu_{ij}^t \tag{3.19b}$$

$$q_{ij}^t,\ k_j^t,\ x_{ij}^t \ge 0 \qquad \text{and the Lagrange multipliers are } \omega_{ij}^t,\ \epsilon_j^t,\ \varpi_{ij}^t \tag{3.19c}$$

where

$$\bar{S}_j^{t+1} = (1-\eta_j)\bar{S}_j^t - \sum_{d_i} q_{ij}^t + k_j^t \tag{3.20a}$$

$$\bar{q}_{ij}^{t+1} = (1-\varphi_{ij})\bar{q}_{ij}^t + x_{ij}^t \tag{3.20b}$$

Since our resource is nonrenewable, we need to subtract the amount of resource depleted in period t to get the available supply capacity in period $t+1$ which is indicated in (3.20a). In this problem, there are $(m \times n) + n$ state variables and $2 \times (m \times n) + n$ choice variables. Unfortunately, solving this problem numerically will be difficult because of the curse of dimensionality[19] as the state space grows

[19]According to Doraszelski and Judd (2012), discrete-time games are limited by their computational burden. In particular, under standard assumptions, there is a curse of dimensionality, since the cost of computing players' expectations over all possible future states increases exponentially in the number of state variables. In the network presented in this dissertation, there are 9 producers and 9 consumers, which makes 90 state and 171 choice variables in each period t before we introduce exogenous uncertainty.

exponentially with both the number of game players and of possible states. Instead, we attempt to derive the first order conditions and the Euler equations for the closed form solution.

The first order conditions with respect to choice variables at time t are

$$q_{ij}^t: \quad \alpha_i - 2\beta_i q_{ij}^t - \beta_i\left(\sum_{k\neq j} q_{ik}^t\right) - \tau_{ij} - \lambda_j^t - \mu_{ij}^t - \omega_{ij}^t + \rho\frac{\partial V(\bar{S}_j^{t+1}, \bar{q}_{ij}^{t+1})}{\bar{S}_j^{t+1}}(-1) = 0 \quad (3.21a)$$

$$k_j^t: \quad -\theta_j + \epsilon_j^t + \rho\frac{\partial V(\bar{S}_j^{t+1}, \bar{q}_{ij}^{t+1})}{\bar{S}_j^{t+1}} = 0 \quad (3.21b)$$

$$x_{ij}^t: \quad -\kappa_{ij} + \varpi_{ij}^t + \rho\frac{\partial V(\bar{S}_j^{t+1}, \bar{q}_{ij}^{t+1})}{\bar{q}_j^{t+1}} = 0 \quad (3.21c)$$

and the first order conditions with respect to state variables at time t:

$$\bar{S}_j^t: \quad \frac{\partial V(\bar{S}_j^t, \bar{q}_{ij}^t)}{\bar{S}_j^t} = \lambda_j^t + \rho\frac{\partial V(\bar{S}_j^{t+1}, \bar{q}_{ij}^{t+1})}{\bar{S}_j^{t+1}}(1-\eta_j) \quad (3.22a)$$

$$\bar{q}_{ij}^t: \quad \frac{\partial V(\bar{S}_j^t, \bar{q}_{ij}^t)}{\bar{q}_{ij}^t} = \mu_{ij}^t + \rho\frac{\partial V(\bar{S}_j^{t+1}, \bar{q}_{ij}^{t+1})}{\bar{q}_j^{t+1}}(1-\varphi_{ij}) \quad (3.22b)$$

Theorem 2: There is unique Cournot-Nash equilibrium in each period t.

Proof: The proof follows from the fact that the Bellman equation of our potential game is strictly concave. $V : \mathcal{R}^{2\times(m\times n)+n} \to \mathcal{R}$. We divide V into three parts since we know that the sum of concave functions is also concave. The first part includes q_{ij}'s only and from the proof of first theorem we know that its Hessian negative definite. The other two parts are linear functions of k_j^t and x_{ij}^t which concludes that they are concave. Our constraints are also linear in terms of choice variables. Therefore, following Zhu (2008), our potential function has a unique equilibrium.

We know that there is a unique equilibrium in each time period t; we need to check the optimal path to the steady state (if it exists).

Next insert (3.21b) and (3.21c) into the first order conditions with respect to the state variables,

$$\frac{\partial V(\bar{S}_j^t, \bar{q}_{ij}^t)}{\bar{S}_j^t} = \lambda_j^t + (\theta_j - \epsilon_j^t)(1 - \eta_j) \tag{3.23a}$$

$$\frac{\partial V(\bar{S}_j^t, \bar{q}_{ij}^t)}{\bar{q}_{ij}^t} = \mu_{ij}^t + (\kappa_{ij} - \varpi_{ij}^t)(1 - \varphi_{ij}) \tag{3.23b}$$

and iterate them by one period

$$\frac{\partial V(\bar{S}_j^{t+1}, \bar{q}_{ij}^{t+1})}{\bar{S}_j^{t+1}} = \lambda_j^{t+1} + (\theta_j - \epsilon_j^{t+1})(1 - \eta_j) \tag{3.24a}$$

$$\frac{\partial V(\bar{S}_j^{t+1}, \bar{q}_{ij}^{t+1})}{\bar{q}_{ij}^t} = \mu_{ij}^{t+1} + (\kappa_{ij} - \varpi_{ij}^{t+1})(1 - \varphi_{ij}). \tag{3.24b}$$

Substitute (3.24a) into (3.22a) and (3.24b) into (3.22b)

$$\frac{\partial V(\bar{S}_j^t, \bar{q}_{ij}^t)}{\bar{S}_j^t} = \lambda_j^t + \rho(\lambda_j^{t+1} + (\theta_j - \epsilon_j^{t+1})(1 - \eta_j))(1 - \eta_j) \tag{3.25a}$$

$$\frac{\partial V(\bar{S}_j^t, \bar{q}_{ij}^t)}{\bar{q}_{ij}^t} = \mu_{ij}^t + \rho(\mu_{ij}^{t+1} + (\kappa_{ij} - \varpi_{ij}^{t+1})(1 - \varphi_{ij}))(1 - \varphi_{ij}). \tag{3.25b}$$

Since we know that (3.23a) is equal to (3.25a) and (3.23b) is equal to (3.25b), we then get

$$\cancel{\lambda_j^t} + (\theta_j - \epsilon_j^t)\cancel{(1 - \eta_j)} = \cancel{\lambda_j^t} + \rho(\lambda_j^{t+1} + (\theta_j - \epsilon_j^{t+1})(1 - \eta_j))\cancel{(1 - \eta_j)} \tag{3.26a}$$

$$\cancel{\mu_{ij}^t} + (\kappa_{ij} - \varpi_{ij}^t)\cancel{(1 - \varphi_{ij})} = \cancel{\mu_{ij}^t} + \rho(\mu_{ij}^{t+1} + (\kappa_{ij} - \varpi_{ij}^{t+1})(1 - \varphi_{ij}))\cancel{(1 - \varphi_{ij})} \tag{3.26b}$$

thus

$$\lambda_j^{t+1} = \frac{(\theta_j - \epsilon_j^t) - \rho(\theta_j - \epsilon_j^{t+1})(1 - \eta_j)}{\rho} \tag{3.27a}$$

$$\mu_{ij}^{t+1} = \frac{(\kappa_{ij} - \varpi_{ij}^t) - \rho(\kappa_{ij} - \varpi_{ij}^{t+1})(1 - \varphi_{ij})}{\rho}. \tag{3.27b}$$

Next we re-date (3.27a) and (3.27b) to period t and insert them into (3.21a) to get the Euler equations for q_{ij}^t:

$$\alpha_i - 2\beta_i q_{ij}^t - \beta_i\left(\sum_{k\neq j} q_{ik}^t\right) - \tau_{ij} - \lambda_j^t - \mu_{ij}^t - \omega_{ij}^t - \rho(\lambda_j^{t+1} + (\theta_j - \epsilon_j^{t+1})(1-\eta_j)) = 0 \quad (3.28a)$$

After rearranging the terms, (3.28a) becomes

$$\alpha_i - 2\beta_i q_{ij}^t - \beta_i\left(\sum_{k\neq j} q_{ik}^t\right) - \tau_{ij} - \mu_{ij}^t - \omega_{ij}^t - \frac{\theta_j - \epsilon_j^{t-1}}{\rho} - \eta_j(\theta_j - \epsilon_j^{t-1}) = 0 \quad (3.28b)$$

Similarly, we get the Euler equations for k_j^t and x_{ij}^t as:

$$k_j^t: \quad -\theta_j + \epsilon_j^t + \rho(\lambda_j^{t+1} + (\theta_j - \epsilon_j^{t+1})(1-\eta_j)) = 0 \quad (3.29)$$

$$x_{ij}^t: \quad -\kappa_{ij} + \varpi_{ij}^t + \rho(\mu_{ij}^{t+1} + (\kappa_{ij} - \varpi_{ij}^{t+1})(1-\varphi_{ij})) = 0 \quad (3.30)$$

For the steady state values, we drop the time superscripts and get

$$\alpha_i - 2\beta_i q_{ij}^\star - \beta_i\left(\sum_{k\neq j} q_{ik}^\star\right) - \tau_{ij} - \mu_{ij}^\star - \omega_{ij}^\star - \frac{\theta_j - \epsilon_j^\star}{\rho} - \eta_j(\theta_j - \epsilon_j^\star) = 0 \quad (3.31a)$$

$$-\theta_j + \epsilon_j^\star + \rho(\lambda_j^\star + (\theta_j - \epsilon_j^\star)(1-\eta_j)) = 0 \quad (3.31b)$$

$$-\kappa_{ij} + \varpi_{ij}^\star + \rho(\mu_{ij}^\star + (\kappa_{ij} - \varpi_{ij}^\star)(1-\varphi_{ij})) = 0 \quad (3.31c)$$

We also know that in the steady state (3.20a) and (3.20b) are going to be

$$\bar{S}_j^\star = (1-\eta_j)\bar{S}_j^\star - \sum_{d_i} q_{ij}^\star + k_j^\star \implies \bar{S}_j^\star = \frac{-\sum_{d_i} q_{ij}^\star + k_j^\star}{\eta_j} \geq 0, \quad (3.32a)$$

which means that in the steady state $k_j^\star \leq \sum_{d_i} q_{ij}^\star$.

$$\bar{q}_{ij}^\star = (1-\varphi_{ij})\bar{q}_{ij}^\star + x_{ij}^\star \implies \bar{q}_{ij}^\star = \frac{x_{ij}^\star}{\varphi_{ij}} \geq 0 \tag{3.32b}$$

In the future, we will solve for the steady state values and also include discrete fixed size capacity investments into the problem.

3.6 Appendix

3.6.1 Two stage capacity investment and production game (Closed-loop Cournot-Nash game)

Proof of Proposition 1: We consider a two stage game where in the first stage producers invest in their supply capacities and in the second stage they choose their production. We assume that there is no uncertainty.

In the second stage producers maximize their profit subject to their supply capacity constraints. Given a graph Q_g and a supply capacity of $\bar{S}_j$, firm j maximizes its profit by choosing q_{ij}.

$$\max_{q_{ij}} \left\{ \sum_{d_i \in N_g(f_j)} \alpha_i q_{ij} - \sum_{d_i \in N_g(f_j)} \beta_i q_{ij} h_i - \sum_{d_i \in N_g(f_j) \backslash d_j} \tau_{ij} q_{ij} \right\} \tag{3.33}$$

$$= \max_{q_{ij}} \left\{ \sum_{d_i \in N_g(f_j)} \alpha_i q_{ij} - \sum_{d_i \in N_g(f_j)} \beta_i q_{ij}^2 - \sum_{d_i \in N_g(f_j)} \beta_i q_{ij} \sum_{f_k \in N_g(d_i) \backslash f_j} q_{ik} - \sum_{d_i \in N_g(f_j) \backslash d_j} \tau_{ij} q_{ij} \right\}$$

subject to

$$\sum_{d_i \in N_g(f_j)} q_{ij} \leq \bar{S}_j = \bar{S}_j^0 + k_j \tag{3.34a}$$

$$q_{ij} \geq 0 \quad \text{for all } \ (i,j) \in g \tag{3.34b}$$

where $\bar{S}_j^0$ is the initial capacity at the beginning of period 0.

$$\mathcal{L} = \sum_{d_i \in N_g(f_j)} \alpha_i q_{ij} - \sum_{d_i \in N_g(f_j)} \beta_i q_{ij} h_i - \sum_{d_i \in N_g(f_j) \setminus d_j} \tau_{ij} q_{ij} + \lambda_j \left(\bar{S}_j - \sum_{d_i \in N_g(f_j)} q_{ij} \right) + \sum_{d_i \in N_g(f_j)} \iota_{ij} q_{ij} \tag{3.35}$$

Then there exists $\lambda_j^\star$ and $\iota_{ij}^\star$ such that $q_{ij}^\star$, $\lambda_j^\star$ and $\kappa_{ij}^\star$ satisfy the following Kuhn-Tucker optimality conditions

$$\frac{\partial \mathcal{L}}{\partial q_{ij}} = \alpha_i - \tau_{ij} - \beta_i \left(\sum_{f_k \in N_g(d_i) \setminus \{f_j\}} q_{ik} + 2q_{ij}^\star \right) - \lambda_j + \iota_{ij} = 0 \tag{3.36a}$$

$$\frac{\partial \mathcal{L}}{\partial \lambda_j} = \bar{S}_j - \sum_{d_i \in N_g(f_j)} q_{ij} \geq 0 \tag{3.36b}$$

$$\frac{\partial \mathcal{L}}{\partial \iota_{ij}} = q_{ij} \geq 0 \tag{3.36c}$$

$$\lambda_j \frac{\partial \mathcal{L}}{\partial \lambda_j} = \lambda_j \left(\bar{S}_j - \sum_{d_i \in N_g(f_j)} q_{ij} \right) = 0 \tag{3.36d}$$

$$\iota_{ij} \frac{\partial \mathcal{L}}{\partial \iota_{ij}} = \iota_{ij} q_{ij} = 0 \tag{3.36e}$$

We get the Cournot-Nash equilibrium[20] flow of $q_{ij}^\star$

$$q_{ij}^\star = \begin{cases} \dfrac{\alpha_i - \tau_{ij} - \lambda_j - \beta_i \left(\sum_{f_k \in N_g(d_i) \setminus \{f_j\}} q_{ik} \right)}{2\beta_i} & \text{if } \frac{\partial \pi_j}{\partial q_{ij}} \Big|_{Q_g} \geq 0 \\ 0 & \text{if } \frac{\partial \pi_j}{\partial q_{ij}} \Big|_{Q_g} < 0 \end{cases} \tag{3.37}$$

in the first stage, producer j chooses his optimal capacity investment

[20] Ilkilic (2010) shows that the unconstrained Cournot game in a bipartite graph has a unique Nash equilibrium.

$$\max_{\bar{k}_j} \Pi_j^\star(q_{ij}^\star) - C(\bar{S}_j) \tag{3.38}$$

Proposition 2: Firm j's profit maximizing supply capacity is obtained by solving $C'(\bar{k}_j) - \lambda_j^\star = 0$.

Proof: Let

$$V_j(\bar{S}_j, \bar{S}_{-j}) = \max_{q_{ij}} \Pi(q_{ij}) \tag{3.39}$$

subject to

$$\sum_{d_i \in N_g(f_j)} q_{ij} \leq \bar{S}_j \tag{3.40}$$

By the Kuhn-Tucker optimality conditions:

$$\lambda_j^\star \left(\bar{S}_j - \sum_{d_i \in N_g(f_j)} q_{ij}^\star \right) = 0 \tag{3.41}$$

Hence,

$$V_j(\bar{S}_j, \bar{S}_{-j}) = \Pi_j^\star(q_{ij}^\star) + \lambda_j^\star \left(\bar{S}_j - \sum_{d_i \in N_g(f_j)} q_{ij}^\star \right) \tag{3.42}$$

By the envelope theorem

$$\frac{\partial V_j(\bar{S}_j, \bar{S}_{-j})}{\partial \bar{k}_j} = \underbrace{\frac{\partial \Pi_j^\star(q_{ij}^\star)}{\partial \bar{k}_j}}_{=0} + \lambda_j^\star \tag{3.43}$$

Hence, $\max_{\bar{S}_j} \Pi_j^\star(q_{ij}^\star) - C(\bar{S}_j)$ is $\lambda_j^\star - C'(\bar{k}_j) = 0$ since $\bar{S}_j = \bar{S}_j^0 + k_j$.

3.6.2 Calibration

In order to quantitatively evaluate different policy scenarios, we first need to calibrate the theoretical model. To calibrate the model parameters, we use the production, consumption, price and trade flow data in 2009. The price data is obtained from international Energy Agency's (IEA) website and other country websites. The data on production, consumption, and trade flows are obtained from BP's Statistical Review of World Energy 2010.

For calibration, we use the first order conditions of our model. The first order conditions with respect to equilibrium flows are same as the ones in the first chapter.[21]

Example: The South American producer labeled as 4, has the objective[22]

$$\max_{q_{14},q_{24},q_{44},k_4} \Pi_4(Q_g) = \max_{q_{14},q_{24},q_{44},k_4} \{p_1 q_{14} + p_2 q_{24} + p_4 q_{44} - \tau_{14} q_{14} - \tau_{24} q_{24} - \theta_4 k_4\} \tag{3.44}$$

subject to

$$q_{14} + q_{24} + q_{44} \leq \bar{S}_4 + k_4 \quad \text{and} \quad q_{14}, q_{24}, q_{44}, k_4 \geq 0 \tag{3.45}$$

By considering the links that carry positive flows[23] in equilibrium, we get the first order conditions as:

$$q_{14}: \quad \alpha_1 - 2\beta_1 q_{14} - \beta_1(q_{11} + q_{15} + q_{16} + q_{17} + q_{18}) - \tau_{14} - \lambda_4 - \iota_{14} = 0 \tag{3.46a}$$

$$q_{24}: \quad \alpha_2 - 2\beta_2 q_{24} - \beta_2(q_{22} + q_{25} + q_{26}) - \tau_{24} - \lambda_4 - \iota_{24} = 0 \tag{3.46b}$$

$$q_{44}: \quad \alpha_4 - 2\beta_4 q_{44} - \lambda_4 - \iota_{44} = 0 \tag{3.46c}$$

$$k_4: \quad -\theta_4 + \lambda_4 + \mu_4 = 0 \tag{3.46d}$$

We assume an interior solution for the capacity constraint,[24] $q^\star_{14} + q^\star_{24} + q^\star_{44} < \bar{S}_4$, this

[21] The reason is that we have the same network with same equilibrium trade flows, production, consumption and price.

[22] For the sake of identification of the problem, we assume that the cost of transporting natural gas to the domestic market is zero.

[23] According to Ilkilic (2010), links that carry zero flows in equilibrium have no role in determining the equilibrium.

[24] We make this assumption only when calibrating the parameters. This assumption is realistic especially in 2009, where due to the global recession, producers had excess supply capacities. When analyzing alternative scenarios we do not impose this assumption.

implies $\lambda_4 = 0$. Therefore, the first order condition in (3.46d) $\implies \theta_4 = \mu_4$ meaning that Kuhn-Tucker condition (3.14g) is satisfied when $k_4 = 0$.[25] We approximate the marginal cost of expanding production capacity by the inverse of reserves[26] to production ratio. If we assume that countries have the same production technologies, then producers with a higher reserves to production ratio must have lower costs of supply capacity expansion. However, our numerical results are sensitive to the choice of this marginal cost parameter.[27]

We apply the same equilibrium condition to each producer from 1 to 9, and get twenty one equations. We also have 9 equations, 1 price equation for each market.[28]

[25]This is because θ_4 is positive and constant.

[26]We obtain proved reserves data from BP's Statistical Review of World Energy, 2010.

[27]If the cost of expanding capacity is sufficiently high, the producer chooses to displace its resources rather than invest in capacity. The resulting outcome would then be the same as in Chapter 1.

[28]Natural gas import prices are usually different for each importer and this price may be different from the domestic producer's price. However, our model assumes that there is a single price of natural gas in each region, which is determined by the total supply of producers connected to that region.

Table 3.1 : Network parameters

	Parameter	Value
Choke price in Europe	α_1	904.27
Choke price in North America	α_2	302.9
Choke price in Asia Pacific	α_3	832.83
Choke price in South America	α_4	260.02
Choke price in West Africa	α_5	220.01
Choke price in North Africa	α_6	199.97
Choke price in Russia	α_7	130.03
Choke price in Middle East	α_8	200.01
Choke price in Australasia	α_9	239.99
Slope of European inverse demand curve	β_1	1.041
Slope of North America's inverse demand curve	β_2	0.184
Slope of Asia Pacific's inverse demand curve	β_3	1.3003
Slope of South America's inverse demand curve	β_4	0.965
Slope of West Africa's inverse demand curve	β_5	10.912
Slope of North Africa's inverse demand curve	β_6	1.445
Slope of Russia's inverse demand curve	β_7	0.134
Slope of Middle East's inverse demand curve	β_8	0.2894
Slope of Australasian inverse demand curve	β_9	1.083
Marginal cost of exporting from South America to Europe	τ_{14}	292.08
Marginal cost of exporting from South America to North America	τ_{24}	148.59
Marginal cost of exporting from West Africa to Europe	τ_{15}	288.85
Marginal cost of exporting from West Africa to North America	τ_{25}	149.43
Marginal cost of exporting from West Africa to Asia Pacific	τ_{35}	311.41
Marginal cost of exporting from North Africa to Europe	τ_{16}	230.02
Marginal cost of exporting from North Africa to North America	τ_{26}	149.07
Marginal cost of exporting from Russia to Europe	τ_{17}	111.41
Marginal cost of exporting from Russia to Asia Pacific	τ_{37}	311.93
Marginal cost of exporting from Middle East to Europe	τ_{18}	273.34
Marginal cost of exporting from Middle East to Asia Pacific	τ_{38}	258.62
Marginal cost of exporting from Australasia to Asia Pacific	τ_{39}	205.18
Europe's marginal cost of supply capacity investment	θ_1	0.054
North America's marginal cost of supply capacity investment	θ_2	0.089
Asia Pacific's marginal cost of supply capacity investment	θ_3	0.038
South America's marginal cost of supply capacity investment	θ_4	0.019
West Africa's marginal cost of supply capacity investment	θ_5	0.006
North Africa's marginal cost of supply capacity investment	θ_6	0.017
Russia's marginal cost of supply capacity investment	θ_7	0.012
Middle East's marginal cost of supply capacity investment	θ_8	0.006
Australasia's marginal cost of supply capacity investment	θ_9	0.021

Table 3.2 : Equilibrium trade flows (in Bcm)

Route	2009	Scenario I	Scenario II	Scenario III	Scenario IV	Scenario V
From Europe to Europe	288.10	282.98	292.33	288.10	290.55	288.10
From North America to North America	813.00	813.00	813.00	813.00	819.23	813.00
From Asia Pacific to Asia Pacific	246.10	246.10	247.32	251.42	248.21	235.93
From South America to Europe	7.60	2.47	11.86	7.60	10.08	7.60
From South America to North America	7.60	7.63	7.59	7.61	14.24	7.61
From South America to South America	134.70	134.70	134.69	134.70	134.69	134.70
From West Africa to Europe	10.70	5.57	14.98	10.70	13.19	10.70
From West Africa to North America	3.10	3.09	3.11	3.08	9.76	3.10
From West Africa to Asia Pacific	6.60	6.60	7.85	11.94	8.74	0.00
From West Africa to West Africa	10.08	10.08	10.08	10.08	10.08	10.08
From North Africa to Europe	67.20	62.07	71.47	67.20	69.68	67.20
From North Africa to North America	5.00	4.99	4.96	5.00	11.61	5.00
From North Africa to North Africa	69.20	69.20	69.19	69.20	69.19	69.20
From Russia to Europe	181.10	175.97	185.38	181.10	183.59	181.09
From Russia to Asia Pacific	6.20	6.20	0.00	11.54	8.34	53.48
From Russia to Russia	485.50	485.44	485.44	485.39	485.39	485.39
From Middle East to Europe	25.60	56.36	0.00	25.60	28.09	25.60
From Middle East to Asia Pacific	47.20	47.20	48.45	52.54	49.34	37.03
From Middle East to Middle East	345.60	345.53	345.54	345.53	345.53	345.54
From Australasia to Asia Pacific	88.30	88.30	89.53	93.63	90.43	78.13
From Australasia to Australasia	110.80	110.80	110.79	110.79	110.79	110.80

Table 3.3 : Equilibrium supply capacity investments (in Bcm)

	2009	Scenario I	Scenario II	Scenario III	Scenario IV	Scenario V
Europe	288.10	0.00	4.23	0.00	2.45	0.00
North America	813.00	0.00	0.00	0.00	6.23	0.00
Asia Pacific	246.10	0.00	1.22	5.32	2.11	0.00
South America	149.90	0.00	4.25	0.00	9.11	0.00
West Africa	30.48	0.00	5.53	5.32	11.29	0.00
North Africa	141.40	0.00	4.22	0.00	9.09	0.00
Russia	672.80	0.00	0.00	5.23	4.52	47.17
Middle East	418.40	30.69	0.00	5.28	4.56	0.00
Australasia	199.10	0.00	1.23	5.33	2.12	0.00

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